# Lec07-approx-TSP

Tuesday, September 12, 2023 11:20 AM

KT Ch. 11 intro. } typically much later topic, but often critical in practice.

What should we do when we don't have an efficient alg.?

Heuristics? Can we guarantee that we aren't that wrong?

## Approximation algorithms

**Def.** Let $A_p(I)$ be the approximate solution value to a min. problem

- instance $I$
- problem $p$ (often dropped in notation)
- alg $A$

Let $OPT_p(I)$ be the optimal (minimum) solution for $I$.

**Ex. goal:** $\forall I, \quad A(I) \le \underbrace{\alpha(|I|)}_{} OPT(I)$

(for any instance $I$)

Some function of problem size $|I|$, want to be small

$\boxed{\text{note: } \alpha \ge 1}$

e.g. $\underbrace{\alpha(n) = 2}_{}$ or $\alpha(n) = \log n$

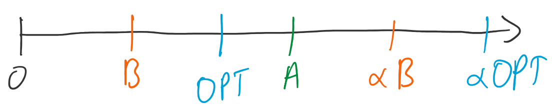
our algorithm never returns a sol more than twice as large as the optimal

## Lower Bounds

How do we analyze how far off from optimal we are without knowing the optimum?

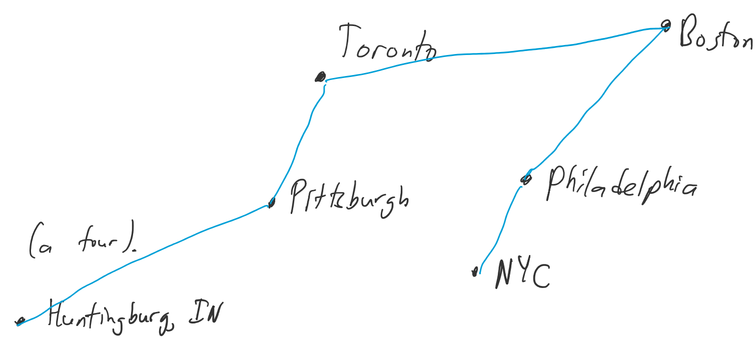
A lower bound is a function $B(I) \le OPT(I) \quad \forall$ instances $I$.

$\Rightarrow$ If $A(I) \le \alpha B(I)$, then $A(I) \le \alpha\, OPT(I)$.




## Euclidean Traveling Salesman Problem (TSP)

Given $n$ cities in $\mathbb{R}^2$ and the Euclidean metric, find the shortest path visiting all of them once (a tour).



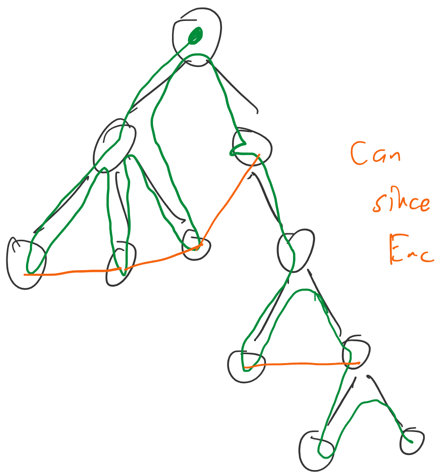

Complete graph with $\binom{n}{2}$ edges

$\boxed{\text{NP-hard problem}}$

## Approximation A:

- Compute a MST $T$.

- Visit the cities in a pre-order traversal = visit a node, then the entire subtree of first child, then subtree of second child, etc.
  (similar to DFS in graph)



Can short-circuit since we are Euclidean

Let $A$ be the edges visited on the tour found by our alg. (counting multiplicity)

Let $A^*$ be edges visited in optimal tour. ~~( " " )~~

Let cost $(S)$ be the total length of edges in multiset $S$.

Thm $\text{cost}(A) \leq 2\,\text{cost}(A^*)$

proof. $\text{cost}(T) \leq \text{cost}(A^*)$ Why? $A^*$ is a tree.

A walk $W$ that traces the MST has length $2\,\text{cost}(T)$ as every edge is crossed twice.

$\Rightarrow \text{cost}(W) = 2\,\text{cost}(T) \leq 2\,\text{cost}(A^*)$

$W$ isn't a tour since it repeats cities.

Short-circuit later visits through the city.

By triangle inequality, this only reduces distance.

$\Rightarrow \text{cost}(A) \leq 2\,\text{cost}(T) \leq 2\,\text{cost}(A^*)$. $\square$

We now have a constant-factor approx alg. for Euclidean TSP.