

# NP-hardness of FUN tasks: video games and Olympic sports

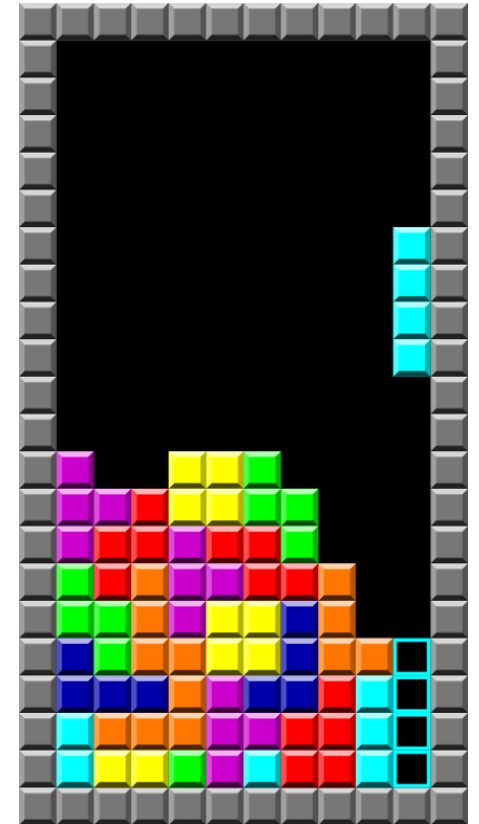
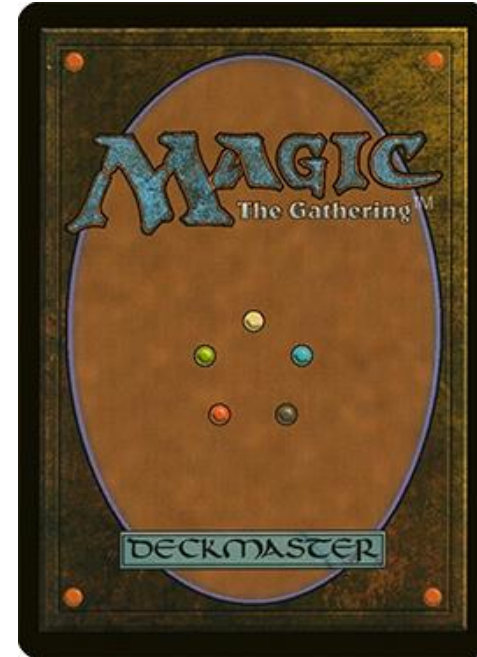
December 8, 2023

15-351/15-650/02-613

Yun William Yu

# TYPICAL GAMES AND COMPLEXITY

- Starting point: a popular game.
- Ingredients:
  - Sequence of decisions that have to be made by the player.
  - A score to be optimized.
- Modifications:
  - Need some natural generalization to allow for arbitrarily large input sizes for complexity and hardness to make sense.
  - May sometimes simplify certain rules or assume rational behavior from players.



A	R	I	S	E
R	O	U	T	E
R	U	L	E	S
R	E	B	U	S



# 3SAT $\leq_P$ SUPER MARIO BROS

- 3SAT Elements:
  - Collection of Boolean variables  $x_1, \dots, x_n$
  - Collection of literals  $t_1, \dots, t_m$ , where each  $t_j = \neg x_i$  or  $t_j = x_i$  for some  $i$ .
  - Collection of clauses  $C_1, \dots, C_k$ , where each  $C_k = t_{j_1} \vee t_{j_2} \vee t_{j_3}$ .
  - Is there a setting of the Boolean variables that makes all clauses true.
- Reduction strategy given in “Classic Nintendo Games are (Computationally) Hard” by Greg Aloupis, Erik Demaine, Alan Guo, and Giovanni Viglietta, 2015. *Theoretical Computer Science*. Part of Special Issue on Fun With Algorithms conference. (2012 arxiv)
  - Question: given a known Mario map, can you finish the level? (path reachability problem)
  - Strategy: force Mario to set variables by choosing a path through the level that reaches particular points – variable are set if you can reach that side of the variable gadget.

# FRAMEWORK FROM [ADGV15]

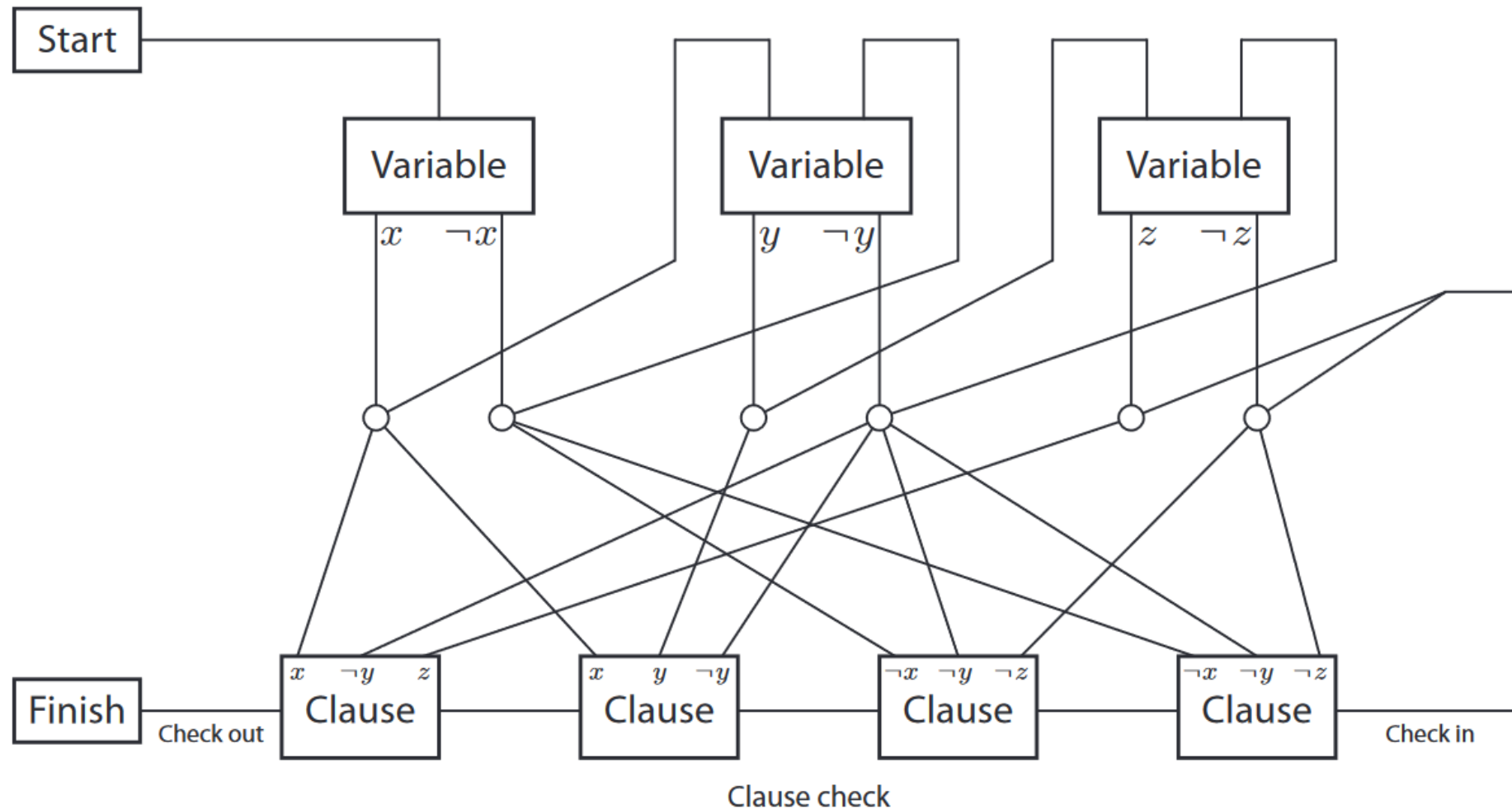
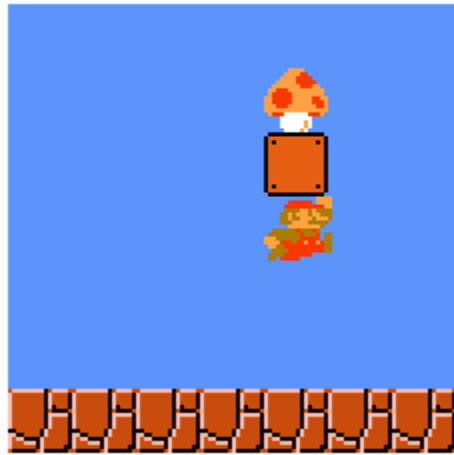
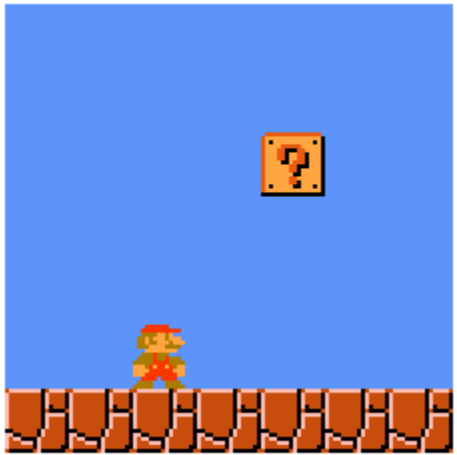


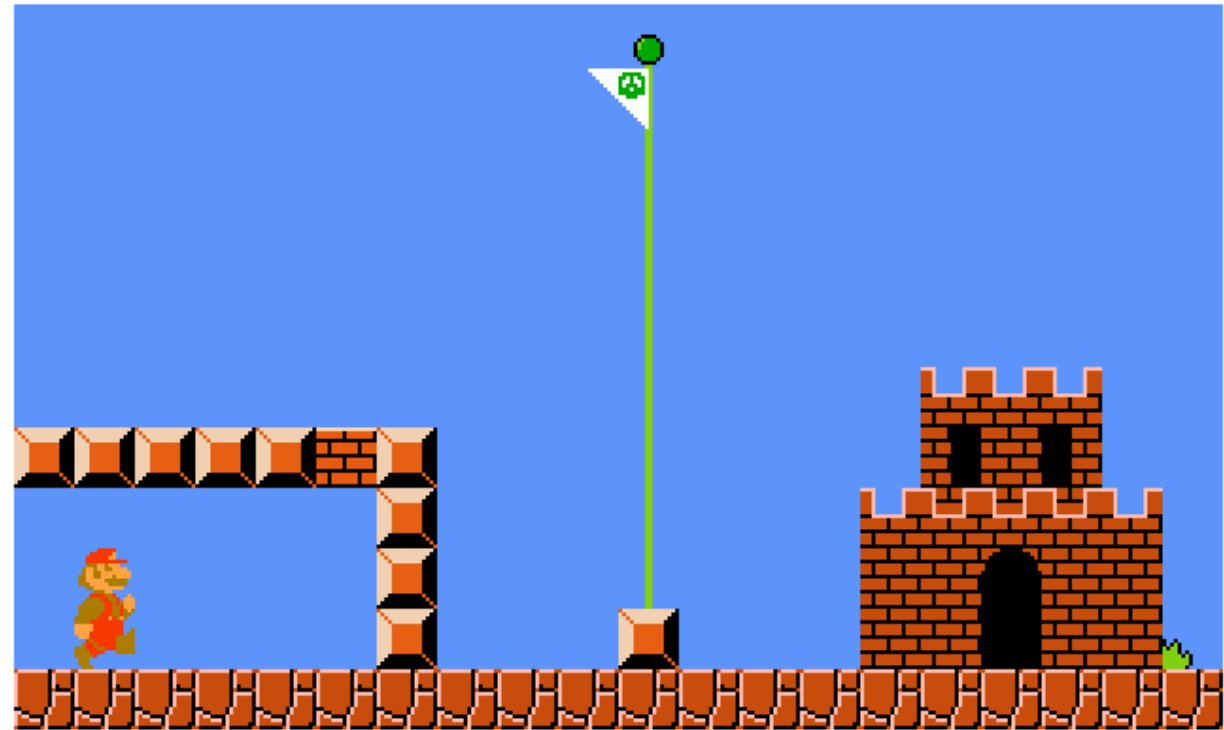
Figure 1: General framework for NP-hardness

# GADGETS

Start Gadget



End Gadget



# VARIABLE GADGET

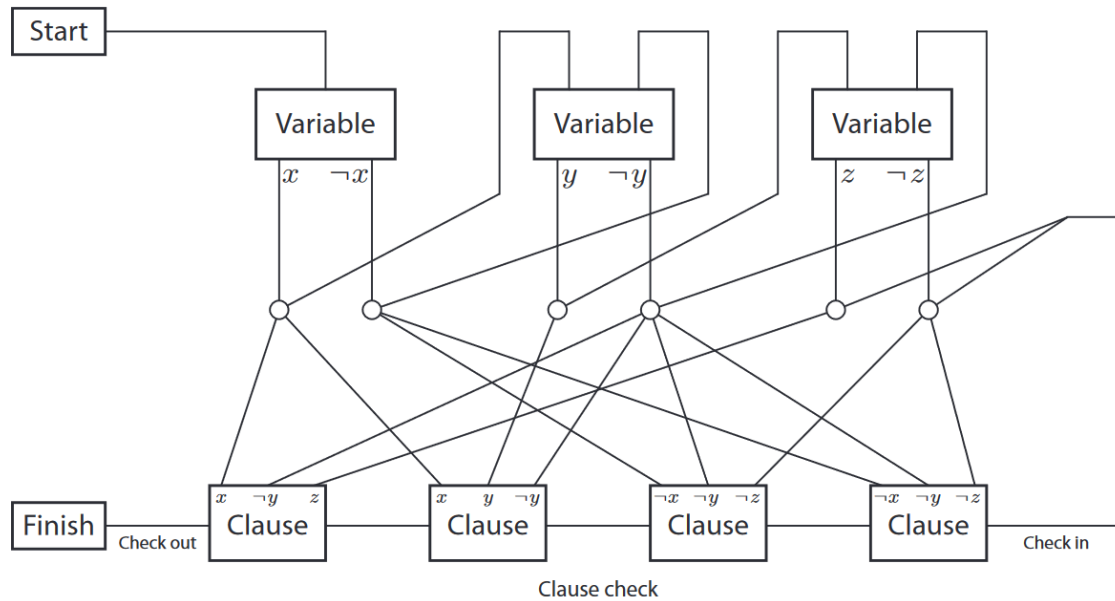
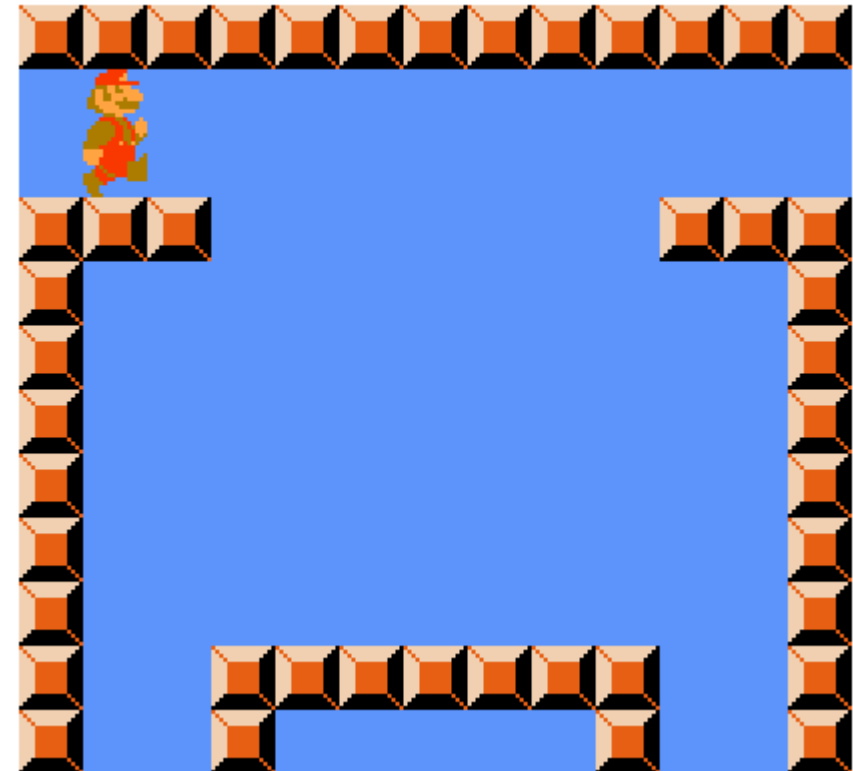


Figure 1: General framework for NP-hardness

## Variable Gadget



# CLAUSE GADGET

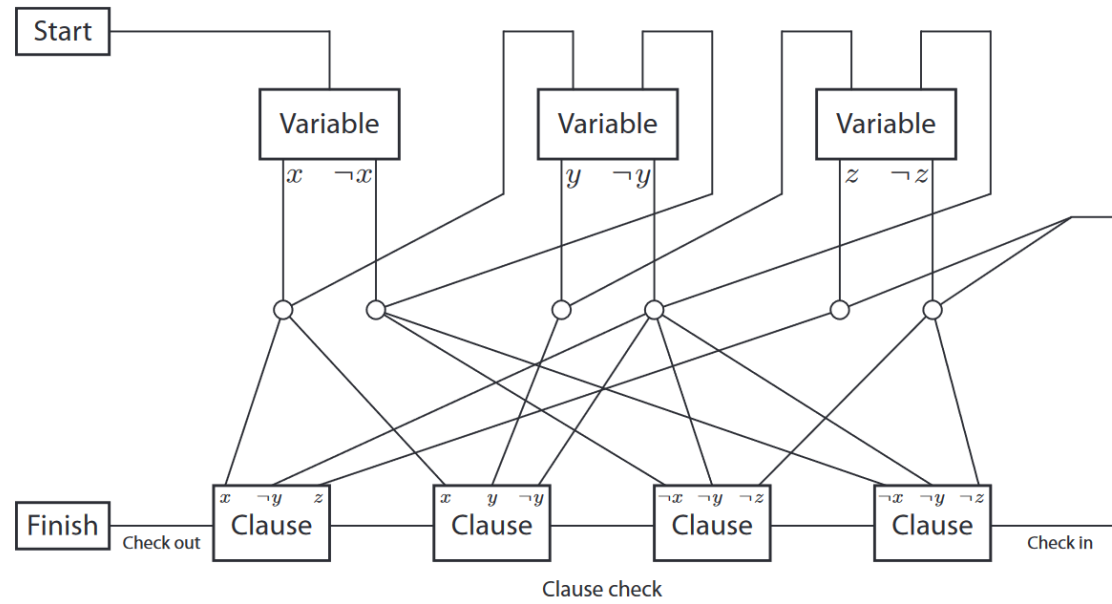
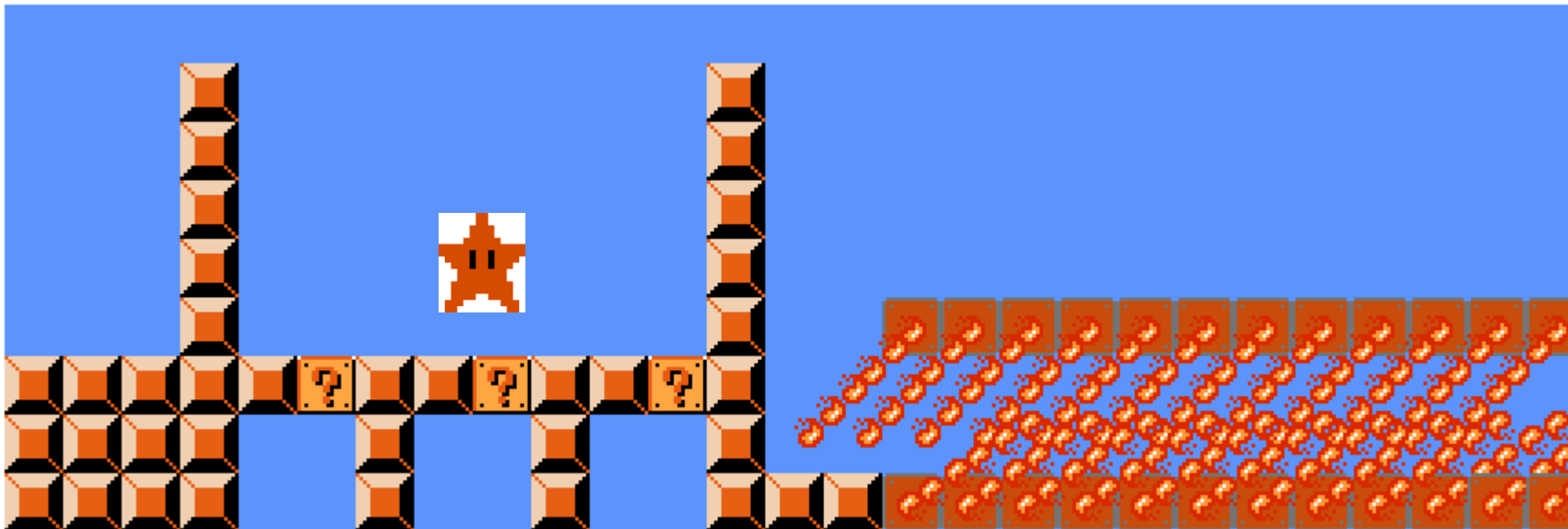


Figure 1: General framework for NP-hardness



# FIXING GRAPH PLANARITY

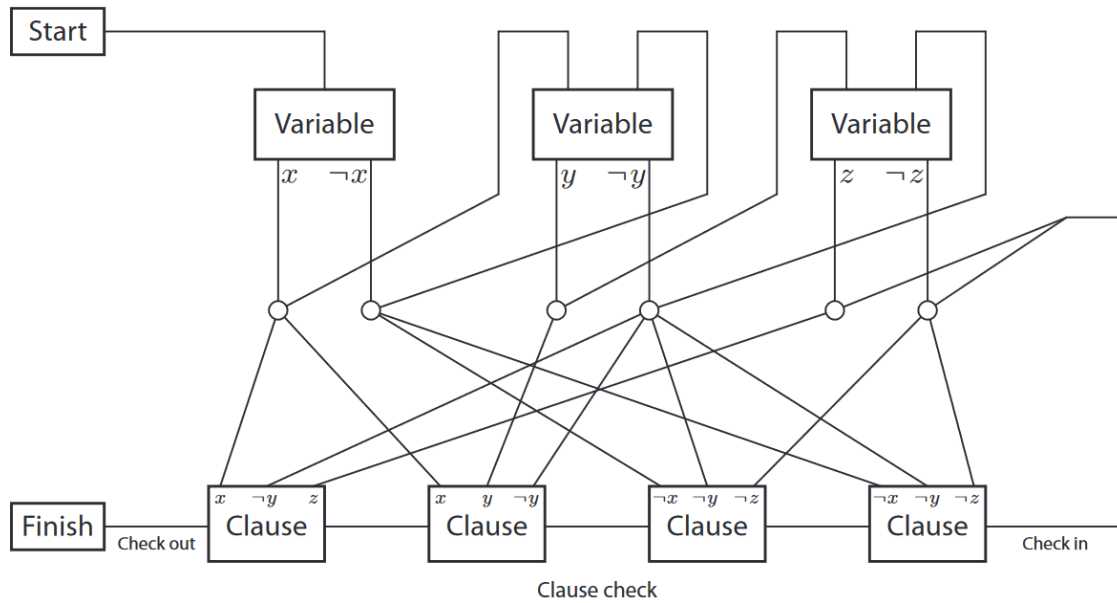
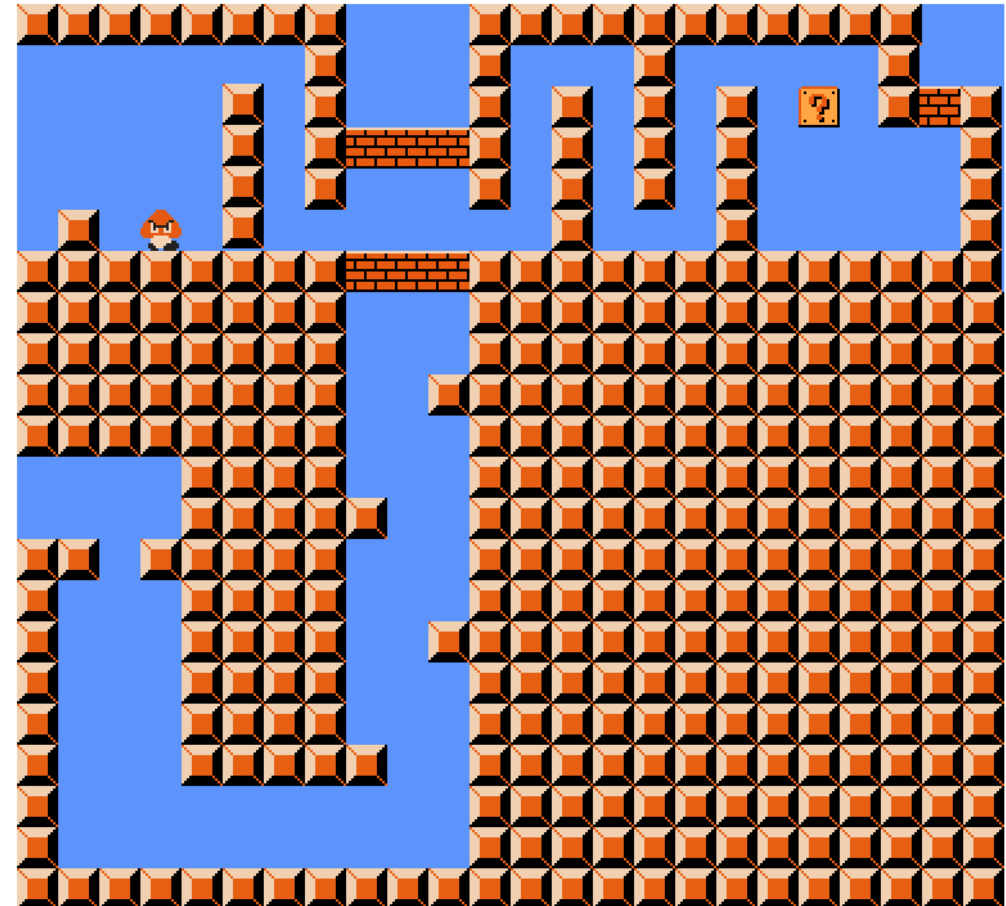


Figure 1: General framework for NP-hardness

## Crossover Gadget





# SUPER MARIO BROS IS PSPACE-COMPLETE

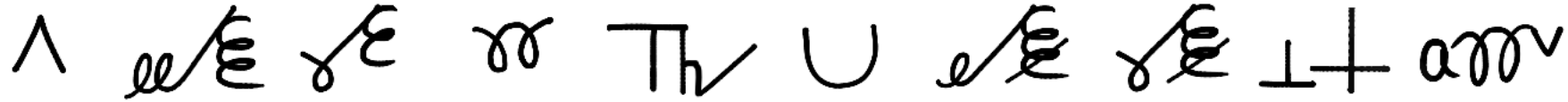
- $NP \subseteq PSPACE$
- Open question about whether they are equal. Most people think they are not.
- Super Mario Bros is PSPACE-complete. Therefore, it is in NP only if  $NP=PSPACE$ .



# STRATEGY IN OLYMPIC SPORTS



# ROUTINES = SEQUENCE OF DECISIONS



- Back handspring
- Back double salto stretched with two turns
- Front salto stretched with full turn
- Front double salto tucked
- Thomas (Arabian) stretched
- Tempo Salto
- Back salto 3/2 turn
- Front salto 3/2 turn
- Split press to Japanese handstand
- Double Arabian pike salto



3/2 salto backwards with 3/2 twists named after Kurt Thomas

Natural generalization to larger input sizes in the form of new moves invented by competitors



# SCORING A ROUTINE – BASIC COMPOSITIONAL SCORE

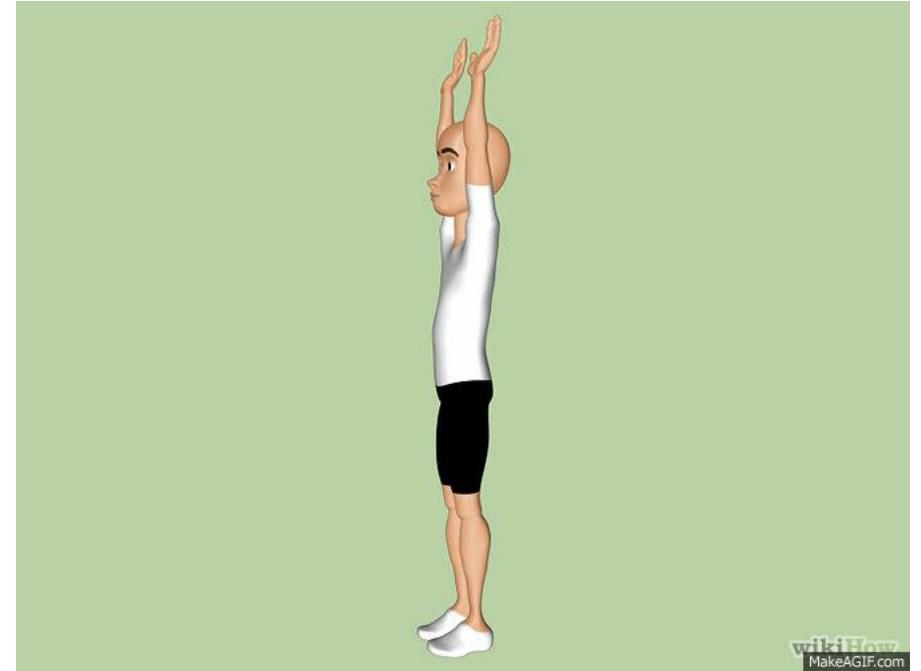
- Compositional scoring assigns each skill a point value, which are added together, as in the below figure skating free skat base value.

#	Executed Elements	Info	Base Value
1	4Lz		11.50
2	4F		11.00
3	4T		9.50
4	3A		8.00
5	CCSp4		3.20
6	StSq4		3.90
7	4T+3T		15.07
8	3Lz+3T		11.11
9	3F+1Eu+3S		11.11
10	ChSq1		3.00
11	FCCoSp4		3.50
12	CCoSp4		3.50
			<b>94.39</b>



# OPTIMAL ROUTINE CONSTRUCTION

- Let's construct a routine:
- You know the following skills:
  - Handstand: 1.0pt
  - Front flip: 1.5pt
  - Back flip: 1.6pt
  - Cartwheel: 0.8pt
  - Splits: 0.9pt
- What is the optimal routine of length 10?



Compositional scoring can lead to degenerate routines.

# 2004 ATHENS GYMNASTICS CONTROVERSY

SUMMER 2004 GAMES -- GYMNASTICS: ALL-AROUND

## *SUMMER 2004 GAMES -- GYMNASTICS: ALL-AROUND; Judges Suspended for Error, But Hamm Will Keep Gold*

The Guardian website header features a dark blue background. On the left, it says "Support the Guardian" in yellow, with "Available for everyone, funded by readers" below it. There are two yellow buttons: "Contribute" and "Subscribe", both with right-pointing arrows. On the right, there are links for "Search jobs", "Sign in", "Search", and "US edition". The Guardian logo is prominently displayed in white, with the tagline "News website of the year" below it. A navigation menu below the logo includes "News", "Opinion", "Sport" (highlighted in blue), "Culture", "Lifestyle", and "More". Below the menu, there are links for "Soccer", "NFL", "Tennis", "MLB", "MLS", "NBA", "NHL", and "F1".

Olympic games  
2004

Uproar in the gym: Russians complain to IOC

Advertisement: "Neither has your support" with a call to action "Support The Guardian" and a right-pointing arrow. The ad has a yellow background and a black border.

NBC Sports banner featuring the NBC Sports logo on the left. In the center is a search bar with a "search site" button. Below the search bar, it says "featuring" followed by links for "Golf on NBC", "NHL on NBC", "Poker", "Fantasy", and "Olympics".

Home Sports / Olympics / Gymnastics

- Scores
- Golf
- NBA
- Baseball
- NFL

## FIG considers gymnastics rule changes

Judges may use video replay in review of start values

Associated Press

updated 5:24 p.m. ET Sept. 24, 2004

MORE SECTIONS

# CODE OF POINTS

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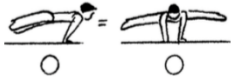
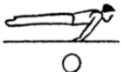

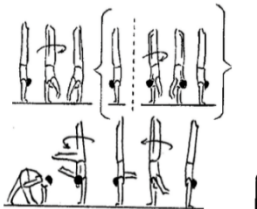
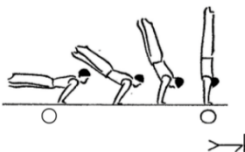
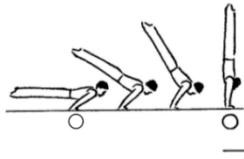
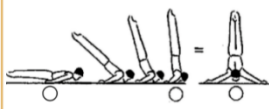



Code MAG 2022

3

170 page-long Men's Artistic Gymnastics  
2022-2024 code of points



# INDEX OF SKILLS IN CODES OF POINTS

A = 0,1	B = 0,2	C = 0,3	D = 0,4	E = 0,5	F = 0,6 G = 0,7 H = 0,8
<b>EG I: Nicht-akrobatische Elemente - Non-acrobatic Elements - Не акробатические элементы.</b>					
<p>25. Stützwaage m. gegr. Beinen (2 s.). Support lever, legs straddle (2 s.). Горизонтальный упор ноги врозь (2 с.).</p> 	<p>26. Stützwaage (2 s.). Support lever (2 s.). Горизонтальный упор (2 с.).</p> 	<p>27. Schwalbe (2 s.). Swallow (2 s.). Самолет (2 с.).</p> 	28.	29.	30.
<p>31. ½ ou 1/1 tour en l'app. renv. ou à l'app. renv. ½ or 1/1 turn in handstand or to handstand. ½ o 1/1 giro en apoyo inv. o al apoyo invertido.</p> 	<p>32. De l'appui facial horizontal ec. (2 s.) s'élever à l'appui renversé (2 s.). From straddled support lever (2 s.) press hdst. (2 s.). Desde el ap. facial horizontal piernas abiertas. (2 s.) elevarse al apoyo invertido (2 c.).</p> 	<p>33. De l'appui facial horizontal 2 s. s'élever à l'appui renversé (2 s.). From support lever 2 s. press handstand (2 s.). Desde apoyo facial horizontal. 2 s. elevarse al ap. inv. (2 c.).</p> 	<p>34. Aus d. Schwalbe (2 s.), Heben i. d. Kreuzhandstand (2 s.). From Swallow 2 s. press to Japanese hdst. (2 s.). Из самолета (2 с.) выход в Японскую стойку (2 с.).</p> <p>(Alvariño)</p> 	35.	36.
<p>37. De l'équerre, éc., tour arr. à la station. From L-sit, etc., turn over bwd. to stand. Desde ángulo piernas abiertas voltear a la posición.</p> 	38.	39.	40.	41.	42.
43.	<p>44. Endorolle gegr. i. d. Handstand (2 s.). Endo roll to handstand (2 s.). Кувырок вперед, сличак (2 с.).</p> 	<p>45. Endorolle geb. i. d. Handstand (2 s.). Gegrätschte Beine in der Endphase möglich. Endo roll piked to handstand (2 s.). Also final phase with straddle. Кувырок вперед, сличак н. вместе (2 с.) Можно разводить ноги без обавки на подъеме в стойку на руках.</p> 	46.	47.	48.
<p>* - Die Anmerkung, dass die Beine in der Endphase gegrätscht werden darf entfällt. * - Does not need to come from handstand to be recognised. * - Могут начинаться и не со стойки на руках.</p>					

# FOUR TYPES OF MODIFICATIONS

1. Anti repetition within a class (possibly overlapping classes)
2. Element group penalty (non-overlapping element groups)
3. Connection bonuses
4. Graph structure / incomplete graphs

## 4. Special repetitions:

- a) Repeated elements (same Code Identification Number) cannot contribute to the “D” score. On Rings, this rule is extended so that a maximum of 1 final strength position in each EG may be recognized for difficulty. Thus, for example only two cross type elements (regular, L cross, or V cross) or support scale type elements (regular or straddled) are permitted in an exercise for difficulty value (one in Group II and one in Group III).
- b) A maximum of 2 Guczoghy type elements can be present in the exercise.

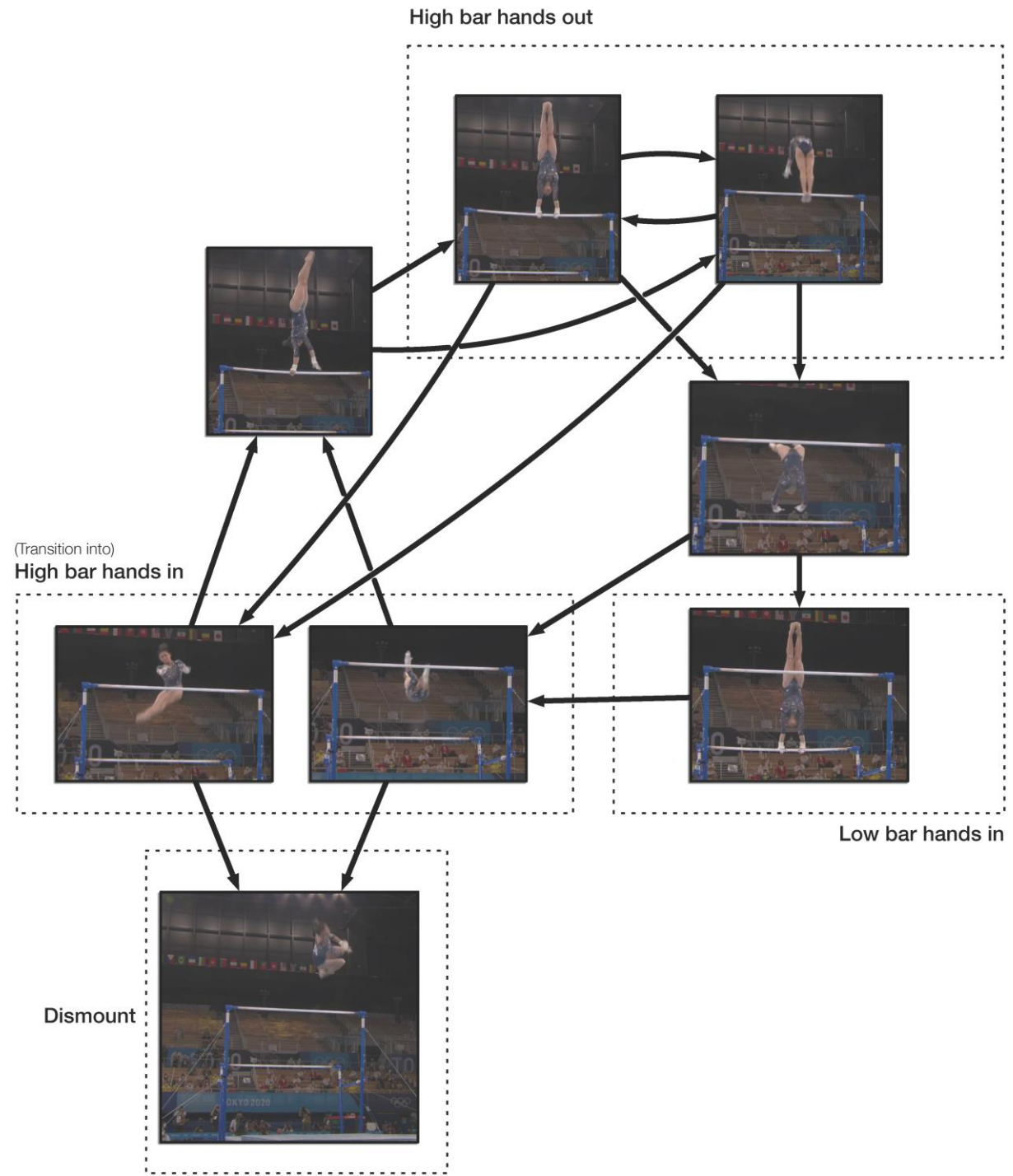
- b) Special rule: Elements to one bar in cross support have the same value as done to two bars, except they increase by one value more when connected to Healy type elements (each Healy element also increases by one value) hold is allowed in the one bar handstand.

## 11.3 Composition Requirements (CR) – D-Panel 2.00

- |   |            |
|---|------------|
| 1. Flight element from HB to LB                             | award 0.50 |
| 2. Flight element on the same bar                           | award 0.50 |
| 3. Different grips ( <i>not cast, MT or DMT</i> )           | award 0.50 |
| 4. Non-flight element with min. 360° turn ( <i>not MT</i> ) | award 0.50 |

# GRAPH STRUCTURE/ INCOMPLETE GRAPHS

- Physical constraints
- Positions
- Momentum
- May differ between athletes, so can be part of the problem instance.



# FORMAL STRUCTURE OF PROBLEM

- Let  $\Sigma$  be the set of possible skills, and  $n = |\Sigma|$ , the number of skills.
  - Let  $m = |S|$  be the maximum length of an allowed routine  $S \in \Sigma^+$ .
  - Let  $q$  be the total number of scoring rules (of any type, defined in the next section).
  - Let  $z = n + m + q$ , the size of the input.
- The input to the problem is the set of skills an athlete can perform, their point values as defined in the Code of Points, the deductions on each skill the athlete will receive, and the total number of additional scoring rules.
  - Decision Problem: given a rational scoring function  $f(S)$  defined by the scoring rules over the set of possible routines, does there exist a routine scoring at least  $X$  points.
  - Optimization Problem: what is the highest scoring routine an athlete can perform?

# SCORING RULES

Basic compositional scoring on a routine  $S = s_1 \dots s_m$  where  $p(s_i)$  is the point value of the skill and  $d(s_i)$  is the deductions for improper form on that skill.

$$f_{\text{BASIC}}(S) = \sum_{i=1}^m (p(s_i) - d(s_i))$$

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<p>Anti-repetition rule <math>(\rho, k)</math> where <math>\rho \subseteq \Sigma</math> and <math>k \in \mathbb{N}</math>. For <math>S = s_1 \dots s_m</math>, define <math>R = r_1 \dots, r_m</math> as a bitstring specifying if each skill is recognized for points and must satisfy Anti-repetition rules.</p>	$f_{\text{ANTI-REPETITION}}(S) = \max_{\text{valid } R} \sum_{i=1}^m (p(s_i) \cdot r_i - d(s_i))$

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<p>Connection rule <math>(s_1, s_2, c_{12})</math> where <math>s_1</math> and <math>s_2</math> are the two consecutive skills and <math>c_{12}</math> is the amount of bonus points to be given.</p>	$f_{\text{CONNECTION}}(S) = \sum_{i=1}^m (p(s_i) - d(s_i)) + \sum_{i=1}^{m-1} c_{s_i, s_{i+1}}$



# SCORING RULES

<p>Basic compositional scoring on a routine <math>S = s_1 \dots s_m</math> where <math>p(s_i)</math> is the point value of the skill and <math>d(s_i)</math> is the deductions for improper form on that skill.</p>	$f_{\text{BASIC}}(S) = \sum_{i=1}^m (p(s_i) - d(s_i))$
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<p>Incomplete Graph rule is an adjacency matrix <math>C</math> of a directed graph. All routines must be paths on the adjacency matrix to score non-zero points.</p>	$f_{\text{INCOMPLETEGRAPH}}(S) = \left( \sum_{i=1}^m (p(s_i) - d(s_i)) \right) \prod_{i=1}^{m-1} c_{s_i, s_{i+1}}$

# EASY REDUCTIONS BETWEEN CLASSES

- IncompleteGraph reduces to Connection because you can encode an unweighted incomplete graph as a weighed complete graph with negative infinite weights on missing edges.
- IncompleteGraph+Connection can still be reduced to just Connection.
- ElementGroup reduces to AntiRepetition because element groups have to be non-overlapping. Thus, we can create a copy of all the skills. Within the copy of the skills, cannot repeat any element group for points, so only makes sense to use the “copied” skill once to get the element group bonus.
- In fact, ElementGroup+AntiRepetition can be reduced to just AntiRepetition.

# (NON)HIERARCHICAL ANTI-REPETITION

► **Definition 3** (*Hierarchical ANTI-REPETITION structure*). Consider a set of ANTI-REPETITION rules  $\{(\rho_1, k_1), \dots, (\rho_q, k_q)\}$ . If there exists a pair  $(\rho_i, \rho_j)$  where  $\rho_i \cap \rho_j \neq \emptyset$  and  $\rho_i \cap \rho_j \neq \rho_i$  and  $\rho_i \cap \rho_j \neq \rho_j$ , then the ANTI-REPETITION rules are *Non-hierarchical*.

- Note that we consider *Non-hierarchical* to imply *Hierarchical* because there is always a subset of the *non-hierarchical* rules that is *hierarchical*.
- We make this distinction because this significantly changes the complexity classes, and very often Codes of Points with even complicated anti-repetition rules are hierarchical.
- *Aside*: we use the distinction *hierarchical* instead of *non-overlapping*, because this allows hierarchical to include prohibiting repeating individual skills, as well as element group bonuses, and a limit on the total number of skills allowed.

# HIERARCHICAL ANTI-REPETITION IS IN P

- We solve this by transforming the problem into a minimum-cost maximum-flow problem, which can be solved in polynomial time via linear programming.
- Encode all of the anti-repetition rules into a tree (possible by hierarchy), with the top-level root corresponding to anti-repetition on the class of all skills (i.e. the length of the routine) and the bottom-level leaves being anti-repetition on individual skills. Each node's capacity is the number of times it can be repeated.
- Use standard in- and out- node duplication trick to convert node capacities into edge capacities for flow problem, and assign a negative weight according to the point values.
- We then introduce a source pointing at the leaves and a target coming from the root.

# NON-HIERARCHICAL ANTI-REP IS NP-HARD

- Reduction from positive one-in-three-SAT (1-in-3-SAT+), where we are given a family of Boolean variables and a collection of triples. The task is to determine if there exists an assignment of the variables such that each triple has exactly one true variable.
- Our reduction encodes each variable as a skill worth 1 point.
- The clauses are encoded as a pair Anti-repetition rules such that each skill within a clause can only be performed once for credit, but also if no skill from a clause is performed, there is a large penalty.
  - Anti-repetition rule with three skills in a similarity class, allowed to repeat once.
  - Anti-repetition rule with all other skills in a similarity class, but allowed to repeat  $m-1$  times, where  $m$  is the length of the routine.
- Routine worth  $m$  points thus gives a solution to 1-in-3-SAT+.

# CONNECTION IS IN P

- Recall that routines are of fixed length  $m$ . We can therefore use dynamic programming.
- Consider fully connected directed graph with weights on the nodes corresponding to skills and weights on the edges corresponding to connection values.
- For any given starting node, can find optimal path of length  $m$  in  $O(n^3m)$  time.
  
- Corollary: IncompleteGraph is also in P.
- Corollary: Connection+IncompleteGraph is in P.

# COMPLEXITY OF RULE CLASSES

	<i>Non-hierarchical</i> ANTI-REPETITION	<i>Hierarchical</i> ANTI-REPETITION	CONNECTION	INCOMPLETE GRAPH
<i>Non-hierarchical</i> ANTI-REPETITION	NP-hard Thm 6			
<i>Hierarchical</i> ANTI-REPETITION		In P Thm 5		
CONNECTION			In P Thm 7	
INCOMPLETE GRAPH				In P Cor 8

# COMPLEXITY OF RULE CLASSES

	<i>Non-hierarchical</i> ANTI-REPETITION	<i>Hierarchical</i> ANTI-REPETITION	CONNECTION	INCOMPLETE GRAPH
<i>Non-hierarchical</i> ANTI-REPETITION	NP-hard Thm 6	NP-hard Thm 6	NP-hard Thm 6	NP-hard Thm 6
<i>Hierarchical</i> ANTI-REPETITION		In P Thm 5		
CONNECTION			In P Thm 7	In P Cor 11
INCOMPLETE GRAPH				In P Cor 8



# HIERARCHICAL ANTI-REP+INCOMPLETE GRAPH IS NP-HARD

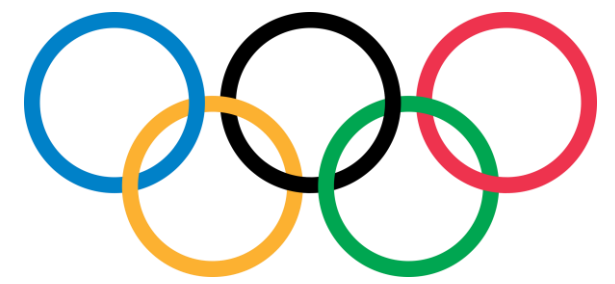
- Reduction from Hamiltonian path, where the goal is to find a simple path through a directed graph such that every node is visited exactly once.
- Encode each node as a skill, with point value 1 and deduction 0. Add an anti-repetition rule for every skill so that we only get credit for each skill once.
- Now let's set  $m=n$ . A routine scoring  $m$  points thus must go through every skill exactly once, while respecting the incomplete graph.
- Corollary: Hierarchical Anti-rep+Connection is also NP-Hard.

# COMPLEXITY OF RULE CLASSES

	<i>Non-hierarchical</i> ANTI-REPETITION	<i>Hierarchical</i> ANTI-REPETITION	CONNECTION	INCOMPLETE GRAPH
<i>Non-hierarchical</i> ANTI-REPETITION	NP-hard Thm 6	NP-hard Thm 6	NP-hard Thm 6	NP-hard Thm 6
<i>Hierarchical</i> ANTI-REPETITION		In P Thm 5		
CONNECTION			In P Thm 7	In P Cor 11
INCOMPLETE GRAPH				In P Cor 8

# COMPLEXITY OF RULE CLASSES

	<i>Non-hierarchical</i> ANTI-REPETITION	<i>Hierarchical</i> ANTI-REPETITION	CONNECTION	INCOMPLETE GRAPH
<i>Non-hierarchical</i> ANTI-REPETITION	NP-hard Thm 6	NP-hard Thm 6	NP-hard Thm 6	NP-hard Thm 6
<i>Hierarchical</i> ANTI-REPETITION		In P Thm 5	NP-hard Cor 10	NP-hard Thm 9
CONNECTION			In P Thm 7	In P Cor 11
INCOMPLETE GRAPH				In P Cor 8



# CHOICE OF SPORTS TO CLASSIFY

- Olympic Sports only, otherwise would have to determine how to classify various styles of competitive dance.
- Some sports were excluded for not having objective Codes of Points:
  - Equestrian (Dressage)
  - Snowboarding (Big Air, Halfpipe, Slopestyle)
- Some sports were excluded for only allowing routines of at most 1-2 skills
  - Diving
  - Vault (in artistic gymnastics)
  - Double mini-trampoline

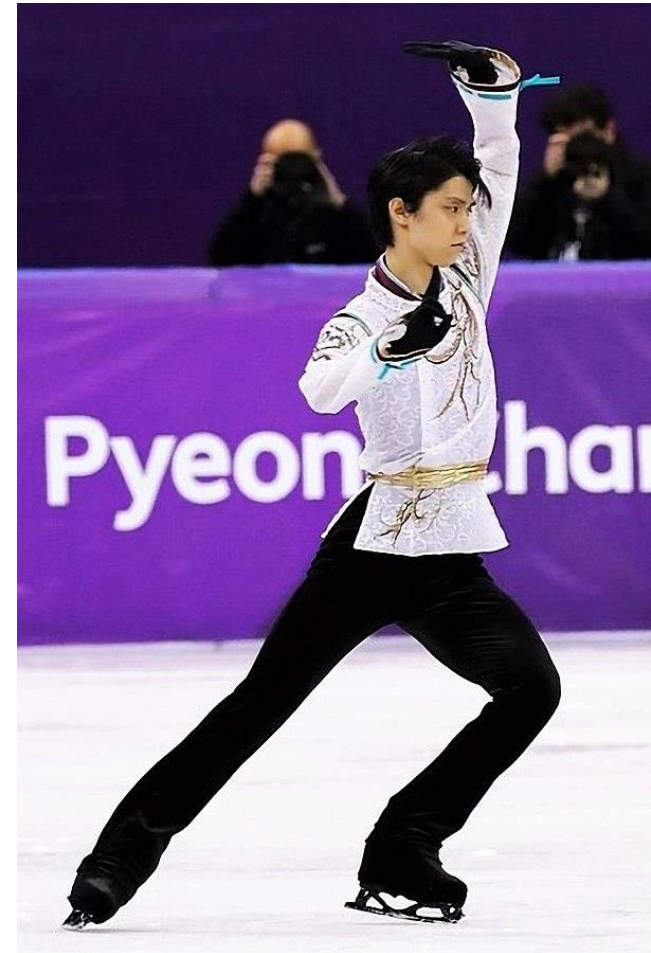
# SKIING

- Most skiing events scored on time, so irrelevant for complexity.
- Left with 5 freestyle events based on aerial tricks:
  - Aerials and Big Air: twists and turns during single jump off ramp
  - Slopestyle and Halfpipe: series of tricks going down a course
  - Mogul: skiing around bumps and performing occasional tricks of ramps (Air component identical to other events)
- There is an Incomplete Graph – e.g. impossible to flip both backwards and forwards in a single jump.
- No real anti-repetition rule – athletes can repeat the same trick
  - Slight complication with Aerials/Big Air because two routines allowed.
  - Athletes “encouraged” to do different tricks in Slopestyle and Halfpipe, but this is not enforced by objective scoring.



# FIGURE SKATING

- Focus on Free Skate (Men's, Women's, Pair)
  - Dance events focus on artistry and disallow difficult tricks.
  - Short Program requires performing 7 required skills, so technically Hamiltonian path, but athletes are allowed to reset position so it's a complete graph.
- We will ignore artistic score and focus only on technical Elements score.
- Jump combinations are treated as higher scoring individual skills, so simply increase the space of skills by a polynomial factor.
- Individual skills may be repeated, but skills within each of four categories can only be repeated a limited number of times. Thus, Hierarchical Anti-Repetition.
- 10% fatigue bonus for skills in second half, but our routine construction rules for Hierarchical Anti-Repetition don't care about order, so can place higher scoring skills later.





# GYMNASTICS

- Lots of different events across rhythmic gymnastics, trampoline, and artistic gymnastics—we only look at the ones with nontrivial routine construction and ignore artistic scores.
- Combinations of anti-repetition, element group, connection, and incomplete graph rules.
- In actuality a lot of additional special case rules that do not fall into those four categories, but luckily does not matter for NP-hardness.



# CLASSIFICATION OF ROUTINE SPORTS


Sport/event	<i>Hierarchical</i> ANTI- REPETITION	<i>Non-hierarchical</i> ANTI- REPETITION	CONNECTION	INCOMPLETE GRAPH	Complexity
Skiing (4 events)	N	N	N	Y	P
Figure skating (Free Skate, Single and Pairs)	Y	N	N	N	P
Rhythmic gymnastics (Individual and Team)	Y	N	(?)	Y	NP-hard
Trampoline	Y	N	N	Y	NP-hard
MAG Floor	Y	N	Y	Y	NP-hard
Pommel Horse	Y	Y	N	Y	NP-hard
Rings	Y	N	N	Y	NP-hard
Parallel bars	Y	N	Y	Y	NP-hard
High Bar	Y	N	Y	Y	NP-hard
WAG Floor	Y	Y	Y	Y	NP-hard
Balance Beam	Y	N	Y	Y	NP-hard
Uneven bars	Y	Y	Y	Y	NP-hard



# HONORABLE MENTION: ARTISTIC SWIMMING

- Technical routine has degrees of difficulty, element groups, etc. However, the set and sequence of skills is fixed for all competitors, so there is no sense of routine construction.
- Free routine and highlight routine allow athletes to construct their own routines, but don't have objective scoring component.
- Proposed 2022 artistic swimming code of points introduces element-based scoring for all events, but not yet in place yet.
- They introduce a new type of scoring rule, the systematic creation of “hybrid” skills. Whether the new rules are NP-hard depends on the interaction of hybrids with the other rules.



 **How score could look, an Example:**

**Event:** Senior Free Duet  
**Requirements:** 6 hybrids + 2 pair acrobatics = 8 elements (EL)

$EI1DD^*Ex + EI2DD^*Ex + \dots + EI8DD^*Ex = \text{Elements Score (EI Score)}$

$EI \text{ score}$ $+ Tr \text{ score}$ $+ Ch \text{ score}$ $+ Mu \text{ score}$ $+ Mp \text{ score}$ $- Sy \text{ errors}$ $- \text{Other penalties}$ <b>Routine Score</b>	$Transitions = Tr \text{ score}$ $Choreography = Ch \text{ score}$ $Musicality = Mu \text{ score}$ $Manner of presentation = Mp \text{ score}$
---	---

# SUMMARY

- Olympic sports can be FUN!
- Attempts to formalize human artistic and subjective judgment into a comprehensive set of rules leads to NP-hard problems.
- Gymnastics is hard! But skiing and figure skating are (computationally) easy.

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Helpful comments on the manuscript and project:

- Aviv Adler
- Quanquan Liu
- MIT Gymnastics Team

2. The Senior Short Program shall consist of the following required elements:

Men

- a) Double or triple Axel Paulsen;
- b) Triple or quadruple jump;
- c) Jump combination consisting of a double jump and a triple jump or two triple jumps or a quadruple jump and a double jump or a triple jump;
- d) Flying spin;
- e) Camel spin or sit spin with only one change of foot;
- f) Spin combination with only one change of foot;
- g) Step sequence fully utilizing the ice surface.

**AS 4.3 Free Routine: Preliminaries / Finals**

Each Solo, Duet, Mixed Duet and Team must perform the Free Routine, which may consist of any listed figures, strokes and / or parts thereof to music.

Free Routines have no restrictions as to the choice of music, content or choreography.

Free Team Routines have a maximum of six acrobatic movements. This does not include partner (2 swimmers) lifts. The acrobatic movement ends with complete submersion of all participants including the one(s) being pushed.

## 7. Special repetitions:










a) A maximum of 2 strength elements (including strength handstands) may be performed in an exercise for content value.

Elements I.1 to I.48 are considered to be strength elements, except:

- Element I.19 - Handstand (2 sec).

- Element I.31 -  $\frac{1}{2}$  or 1/1 turn in handstand or to handstand.

b) A maximum of 2 circle, flair or Russian elements may be performed in an exercise for difficulty value.

Difficulty Components							
<b>Difficulty of Body (DB)</b> Highest 9 counted	<b>Difficulty of Apparatus (DA)</b>  Minimum 1 Maximum 20 (in performance order)						
<b>Special Requirement</b>							
<table border="1"> <tr> <td style="text-align: center;"> <b>Difficulty of Body Groups:</b> </td> <td style="text-align: center;"> <b>Full body waves (W)</b> </td> </tr> <tr> <td>           Jump/Leaps  Minimum 1         </td> <td rowspan="3" style="text-align: center;">           Minimum 2         </td> </tr> <tr> <td>           Balances  Minimum 1         </td> </tr> <tr> <td>           Rotations  Minimum 1         </td> </tr> </table>		<b>Difficulty of Body Groups:</b>	<b>Full body waves (W)</b>	Jump/Leaps  Minimum 1	Minimum 2	Balances  Minimum 1	Rotations  Minimum 1
<b>Difficulty of Body Groups:</b>		<b>Full body waves (W)</b>					
Jump/Leaps  Minimum 1	Minimum 2						
Balances  Minimum 1							
Rotations  Minimum 1							
<b>Dynamic Elements with Rotation - R</b>  <b>Maximum 5</b> (in performance order)							

d) Backward swings to handstand that simply reverse direction and swing back down in the reverse direction are deducted (composition errors) each time with 0,30 points.

Specific examples of such layaways are:

- Following a kip cast or back uprise to handstand - layaway to giant swing bwd., Stalder, free hip, ½ turn to el-grip, etc.
- Following a backward uprise to handstand and hop to overgrip - swing forward to giant swing bwd., Stalder, free hip, etc.

(Also other angle deductions also need to be applied for missing the handstand position).

e) Any flight element with salto over the bar requires a giant swing afterward or -0.3 (E-jury deduction).