Lec06-graph-traversal-applications

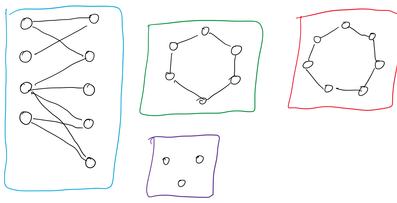
Thursday, September 3, 2026 3:51 PM

Applications

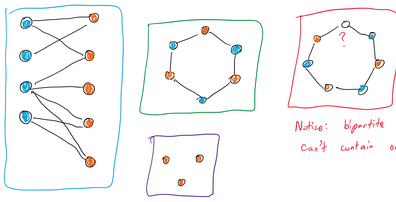
KT 3.4 / 3.6

A graph is **bipartite** if ∃ a two-coloring

i.e. if you can divide the nodes into two colors such that all edges connect nodes of different colors



Question: Which of these are not bipartite?



Notice: bipartite graphs can't contain odd cycles

How to test bipartiteness?

Do a BFS from any node, swapping colors at each layer.

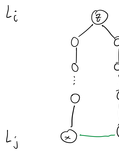
Check if any edge is monolayer. (recall edges are always either between adj layers or the same layer)

proof of correctness

1. If there are no monolayer edges, then every edge is b/t adj layers, which can be colored opposite.
2. If ∃ edge $(x,y) \in G$ on the same layer L_j, let $z \in L_i$, $i < j$ be the least common ancestor of x, y in BFS tree.

   $\Rightarrow$ $z - x - y - z$ is a cycle of length $2(j-i)+1$, odd

   $\Rightarrow$ G is not bipartite.

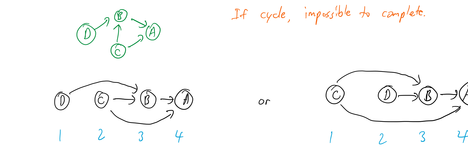


A **DAG** (directed acyclic graph) is a directed graph that contains no (directed) cycles. (after leaving a node, you can never return)

Ex Project Dependencies — How do you order jobs so that when a job is started, all its dependencies are done?

Task A needs B & C. B needs C & D.

If cycle, impossible to complete.



Topological sort: Given a DAG $G = (V, E)$, find a bijective mapping f from V to $\{1, \ldots, |V|\}$ s.t. $\forall (u,w) \in E$, $f(u) < f(w)$.

Thm Every DAG has a node with no incoming edges. (i.e. possible start points)

proof. Suppose not.

Then keep following edges backward, and in $\leq n+1$ steps, will have repeated a node $\Rightarrow$ cycle $\Rightarrow$ contradiction

Topo sort alg 1:

Let $i = 1$

While $i \leq |V|$

Find node u w/o incoming edges

Set $f(u) = i$

Delete u from graph

i++

Implementation

Array Income[w] = # incoming edges for w.

List S of nodes w/o incoming edges.

{ decrement Income[v] for all neighbors. If Income[w] = 0, add w to S.

Works because after removing u, still a DAG since we didn't add cycles, so we can't get stuck.

Running Time:

Initialize Income[w] by counting edges via DFS — $O(|V|+|E|)$

{ Loop happens $|V|$ times

{ Each neighbor decrement takes $O(1)$ time, & total # times we encounter neighbors is $O(|E|)$

$O(|V|+|E|)$ time.

Topo sort alg 2: (via DFS finishing times)

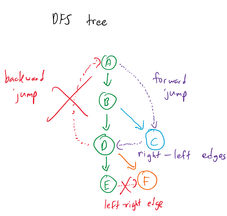
Given a DFS, can associate 2 numbers w/ every node.

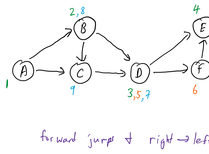
discovery time: d[u] = time at which node u is first visited

finishing time: f[u] = time at which u and all neighbors are fully explored (last visit)

Clearly, $d[u] \leq f[u]$

DFS tree





forward jumps & right → left edges allowed.

backward jump can't exist, because would create a cycle & we are in a DAG

left → right edge can't exist, because the DFS would have used it in the "down" subtree.

Class question: Which types of edges are allowed?

Algorithm

Every edge (u,v) in a DAG has $f[v] < f[u]$ (deeper parts of tree finish first)

Reverse ordering by finishing times gives a topo sort.