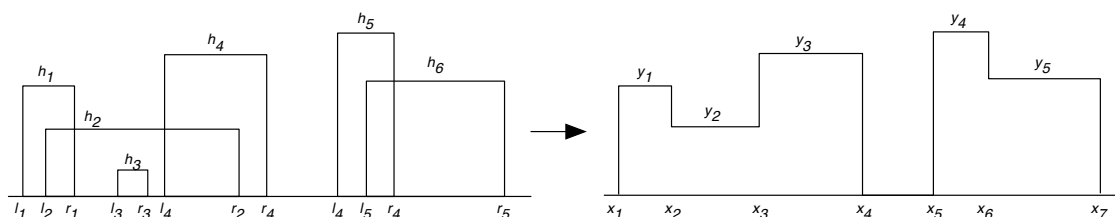


15-351 / 15-650 / 02-613: Exercises 1c

1. Design an $O(\log n)$ -time algorithm that takes a real number a and a positive integer n as input and finds a^n . All operations you can use is addition, subtraction, multiplication, division, and rounding a real number up/down to an integer, all of which takes $O(1)$ time. You are not allowed to use the logarithm or exponentiation, because these two operators are not defined on all groups.

Hint: The result a^n might be very large or small, but you do not need to worry about it. Also, for any $m \in \{1, \dots, n-1\}$, we have $a^n = a^{n-m}a^m$.

2. You want to draw a two-dimensional skyline like the following:



You are given a list of the building x -coordinates and their heights:

$$(l_1, h_1, r_1), (l_2, h_2, r_2), \dots, (l_n, h_n, r_n)$$

This list will be sorted in increasing order of left-most x -coordinate. Design an $O(n \log n)$ algorithm to produce the skyline line to draw in the format:

$$x_1, y_1, x_2, y_2, x_3, \dots$$

meaning that at x_1 we draw a building at height y_1 until x_2 at which point we draw a line up or down to high y_2 and then continue horizontally until x_3 and so on. Note that for this problem, you need only *sketch* the main ideas without providing detailed discussion for algorithm correctness.