15-351 / 15-650 / 02-613: Exercises 3b

For the following NP-completeness proof problems, you can only assume that Vertex Cover, Set Cover, Independent Set, 3-SAT, Hamiltonian Cycle, and Hamiltonian Path, Traveling Salesman Problem, which we discussed in class, are NP-complete.

1. Let Integer Linear Programming be the decision problem asking whether a given maximization integer linear program has a solution of objective value $\geq$ a given k. Prove Integer Linear Programming is NP-complete.

2. A double-Hamiltonian traversal in an undirected graph G is a closed walk that goes through every vertex in G exactly *twice*.

   Prove that the problem of testing whether a graph has a double-Hamiltonian traversal is NP-complete.

3. Suppose $G = (V, E)$ is an undirected graph. A *strongly independent set* is a subset S of vertices such that for any two vertices $u, v \in S$ there is no path of length ≤ 2 between u and v. Consider the following Strongly Independent Set problem: Given an undirected graph $G = (V, E)$ and an integer k, does G have a strongly independent set of size k?

   Prove that the Strongly Independent Set problem is NP-complete.

4. The Subgraph-Isomorphism Problem takes two undirected graphs G_1 and G_2 and asks whether G_1 is a subgraph of G_2. In other words, we ask whether there is an injective function f to map the vertices of G_1 to the vertices of G_2 such that there is an edge $\{u, v\}$ in G_1 exactly when $\{f(u), f(v)\}$ is in G_2.

   Prove that Subgraph-Isomorphism Problem is NP-complete.

5. The Required Paths Subgraph problem takes as input an undirected graph G with nodes $v_1, \ldots, v_n$, an $n \times n$ symmetric matrix R of natural numbers, and an integer b, and asks "Is there a set S of b edges of G with the following property: between every pair of nodes v_i and v_j, $i \neq j$, there are at least R_{ij} disjoint paths (that is, paths sharing no other node except for the endpoints) using edges in S."

   Prove that this problem is NP-complete.