
1: Asymptotic Analysis

1. Complete the table below to satisfy the following statement: For functions f and g , we write $f(n) = \mathbf{A}$ if there exists constants $C > 0$ and $n_0 > 0$ such that for all $n \geq n_0$, $\mathbf{B}$ is true. Options: $O(g(n))$, $\Theta(g(n))$, $\Omega(g(n))$.

A	B
	$f(n) \leq C(g(n))$
	$f(n) \geq C(g(n))$
	$c(g(n)) \leq f(n) \leq C(g(n))$

2. What is the time complexity of the example function below?

```

1 def example_function(arr):
2     n = len(arr)
3     total = 0
4     for i in range(n):
5         for j in range(10):
6             total += arr[i] * j
7     return total

```

3. What is the time complexity of the example function below?

```

1 def tell_a_friend(n):
2     if n > 0:
3         for i in range(2):
4             tell_a_friend(n - 1)

```

4. Rank the following functions from lowest to highest in terms of asymptotic growth:

(a) $f(n) = n^2$

(b) $g(n) = 2^n$

(c) $h(n) = \log(n)$

(d) $k(n) = n!$

(e) $m(n) = n \log(n)$

5. Show the following statement is false: Given two algorithms A and B which are known to have time complexities of $O(n)$ and $O(\log n)$ respectively, algorithm B will always be faster than algorithm A.
6. Show that $4n^2 + 2000n + 8$ has a time complexity equal to $O(n^2)$.
7. Show that for any constant $b > 2$, $\log_b(n) = \theta(\log n)$

2: Solutions

Asymptotic Analysis

	A	B
1.	$O(n)$	$f(n) \leq C(g(n))$
	$\Omega(g(n))$	$f(n) \geq C(g(n))$
	$\Theta(g(n))$	$c(g(n)) \leq f(n) \leq C(g(n))$

2. $10n = O(n)$

3. 2^n

4. $\log(n) < n \log(n) < n^2 < 2^n < n!$

5. One simple counterexample to this statement would be if function A was equal to $0.0001n$ and function B was equal to $1000 \log_2(n)$. For $n = 10$, the functions would have runtimes equal to 0.001 and 3320 respectively. In this case, function A is significantly faster, even though A and B are $O(n)$ and $O(\log(n))$ respectively.

6. To solve this, we need to find the upper bound of the function $C * n^2$ such that C is a constant for all possible values of $n \geq n_0$. Here is an example:

$$n_0 = 1 \quad (1)$$

$$4n^2 > 4n^2 \quad (2)$$

$$2000n^2 > 2000n \quad (3)$$

$$8n^2 > 8 \quad (4)$$

$$2012n^2 > 4n^2 + 2000n + 8 \quad (5)$$

So we found a constant $C = 2012$, such that $Cn^2 > 4n^2 + 2000n + 8$, for all $n \geq n_0$

7. We can use the change of base formula to convert any $\log_b(n)$, where $b > 1$, to $\log(n)$ as follows:

$$\log_b(x) = \frac{\log_k(x)}{\log_k(b)} \quad (6)$$

We use this formula to provide $\log_b(n)$ with two constants that make it $\theta(\log(n))$