Problem Set 8

[Your name] and [student ID]
MAT1801-2020

Problem 1 [BHK 11.8] (10 points). If $f(x)$ is frequency limited by 2π, prove that

$$f(x) = \sum_{k=0}^{\infty} f(k) \frac{\sin(\pi(x-k))}{\pi(x-k)}.$$

Hint: Use the Nyquist sampling theorem which states that a function limited by 2π is completely determined by samples spaced one unit apart. Note that this means that

$$f(k) = \int_{-\infty}^{\infty} f(x) \frac{\sin(\pi(x-k))}{\pi(x-k)} dx$$

Problem 2 [BHK 11.9] (10 points). Compute an approximation to the scaling function that comes from the dilation equation

$$\phi(x) = \frac{1+\sqrt{3}}{4}\phi(2x) + \frac{3+\sqrt{3}}{4}\phi(2x-1) + \frac{3-\sqrt{3}}{4}\phi(2x-2) + \frac{1-\sqrt{3}}{4}\phi(2x-3)$$

Please do not use a pre-existing wavelet transform or dilation equation library. i.e. implement a numerical algorithm to approximate the solution. The book provides several example algorithms, but you may implement the algorithm of your choice, so long as you reference/describe your alogorithm of choice.

Problem 3 [BHK 11.12] (10 points). Prove that if the scale functions defined by a dilation equation are orthogonal, then the sum of the even coefficients must equal the sum of the odd coefficients in the dilation equation. That is, $\sum_k c_{2k} = \sum_k c_{2k+1}$.