Problem Set 7

[Your name] and [student ID]
MAT1841-2021

Problem 1 [BHK 11.9] (25 points). Compute an approximation to the scaling function that comes from the dilation equation

$$\phi(x) = \frac{1+\sqrt{3}}{4}\phi(2x) + \frac{3+\sqrt{3}}{4}\phi(2x-1) + \frac{3-\sqrt{3}}{4}\phi(2x-2) + \frac{1-\sqrt{3}}{4}\phi(2x-3)$$

Please do not use a pre-existing wavelet transform or dilation equation library. i.e. implement a numerical algorithm to approximate the solution. The book provides several example algorithms, but you may implement the algorithm of your choice, so long as you reference/describe your alogorithm of choice.

Problem 2 [BHK 11.12] (25 points). Prove that if the scale functions defined by a dilation equation are orthogonal, then the sum of the even coefficients must equal the sum of the odd coefficients in the dilation equation. That is, $\sum_k c_{2k} = \sum_k c_{2k+1}$.

Problem 3 [BHK 6.16] (25 points). Suppose we want to pick a row of an $n \times n$ matrix at random where the probability of picking row i is proportional to the sum of squares of the entries of that row. How would we do this in the streaming model?

1. Do the problem when the matrix is given in row order. i.e. you receive a stream of the entries of the matrix one-by-one, but you get all the entries of row 1 first, and then all the entries of row 2, etc.
2. Do the problem when the matrix is given in column order. i.e. you receive a stream of the entries of the matrix one-by-one, but you get all the entries of column 1 first, and then all the entries of column 2, etc.
3. Do the problem when the matrix is represented in sparse notation: it is just presented as a list of triples (i, j, a_{ij}) in arbitrary order.

Suppose you are limited to storing at most $O(n)$ entries at a time (i.e. you can store several rows of the matrix if needed, but not the entire matrix). You may need multiple passes for some of the variants above, but try to give a solution in the minimum number of passes. Here, a pass means that you restart the stream a second, or third time. Hint: we didn't discuss the streaming sampling algorithm much in class, but this is covered in some depth in the book: sections 6.1 and 6.3.2.

Problem 4 [Project Progress Report] (25 points). Problem 4 is not so much a problem as it is a progress report. Please write a 1-page summary of your progress on your project so far, including any dead-ends you've hit, twists and turns you've had to take, and results so far.