

The number line

Lecture 1b: 2022-01-12

MAT A02 – Winter 2022 – UTSC

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What are y'all studying?

Think like a mathematician

- What problem remains after having invented negative numbers?

- A: We don't know how to subtract negative numbers
- B: We don't know how to add negative numbers
- C: We don't know how addition and subtraction interact
- D: All of the above
- E: None of the above

$$-4 - (-3) =$$

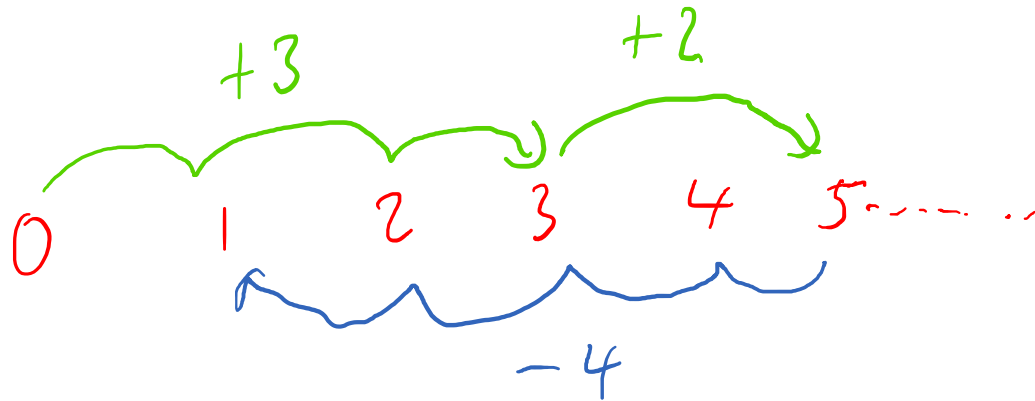
$$-4 + (-3) =$$

$$4 - 3 + 5$$

The positive number line

- We can arrange positive numbers on a line.
- The addition is moving to the right, subtraction to the left.

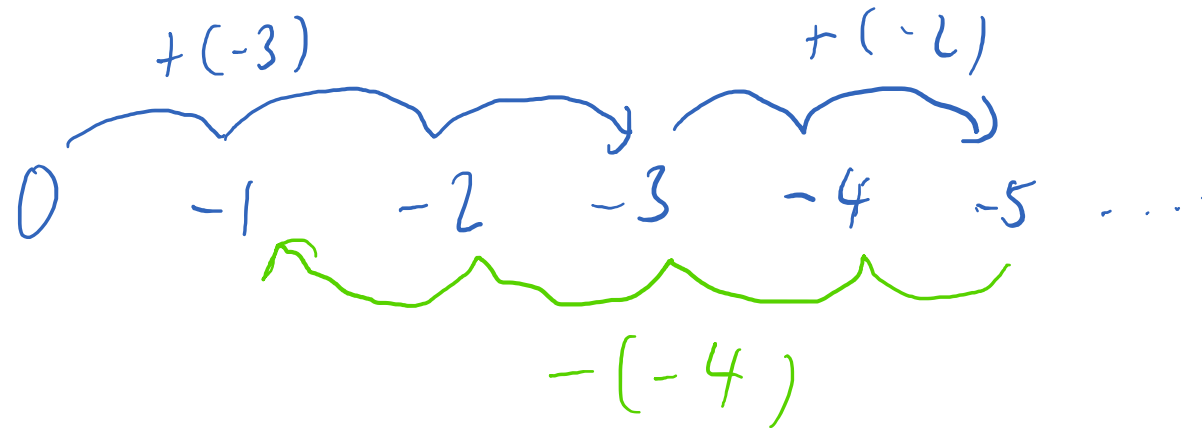
$$0 + 3 + 2 - 4$$



The negative number line

- The negative numbers are a copy of the positive numbers, so adding two negative numbers together should behave the same
- Similarly, subtracting two negative numbers should be similar

$$0 + (-3) + (-2) - (-4) = -1$$



Combining the two number lines

- Can we combine the positive and negative number lines together? If so, how?

A: Stack on top of each other
B: Side-by-side
C: Side-by-side but flip one
D: One horizontal, one vertical
E: No, they cannot be combined

- What facts can we use to combine the two lines together?

0 1 2 3 4 ...

-1

-2

-3

-4

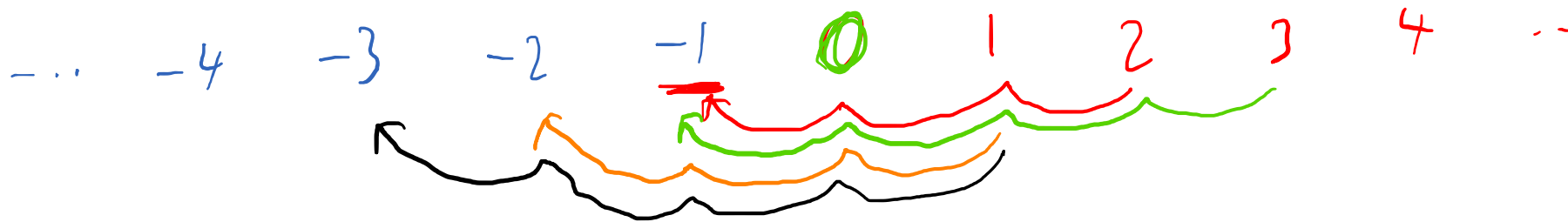
...

A: Position of the 0
B: Definition of subtraction
C: The fact that they are copies
D: Extra dimensions
E: None of the above

The number line

- Remember, 0 is present in both lines, and subtraction of two positive numbers can give a negative number.

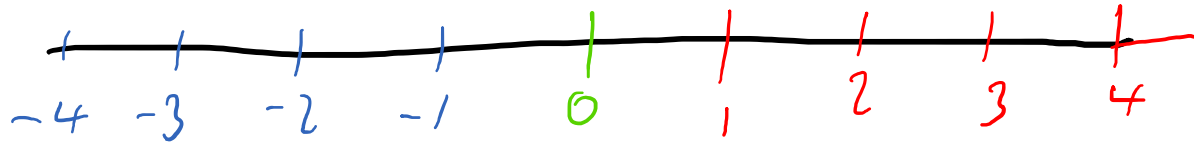
$$\begin{array}{ll} 2-3 = -(3-2) = -1 & 1-3 = -(3-1) = -2 \\ 3-4 = -(4-3) = -1 & 1-4 = -(4-1) = -3 \end{array}$$



- Because of the way subtraction is defined, the negative copy of number line has to be flipped.

The number line

- Let's write the negative and positive numbers on a long line, with negative numbers to the left and positive numbers to the right:



- Another way to understand addition of positive numbers is how far to the right we are moving along the number line.

$$2 + 3 = \text{move right } 2 \quad \left. \begin{array}{l} \text{move right } 3 \\ \end{array} \right\} \text{if } \rightarrow \text{ is a right move}$$

$\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow$

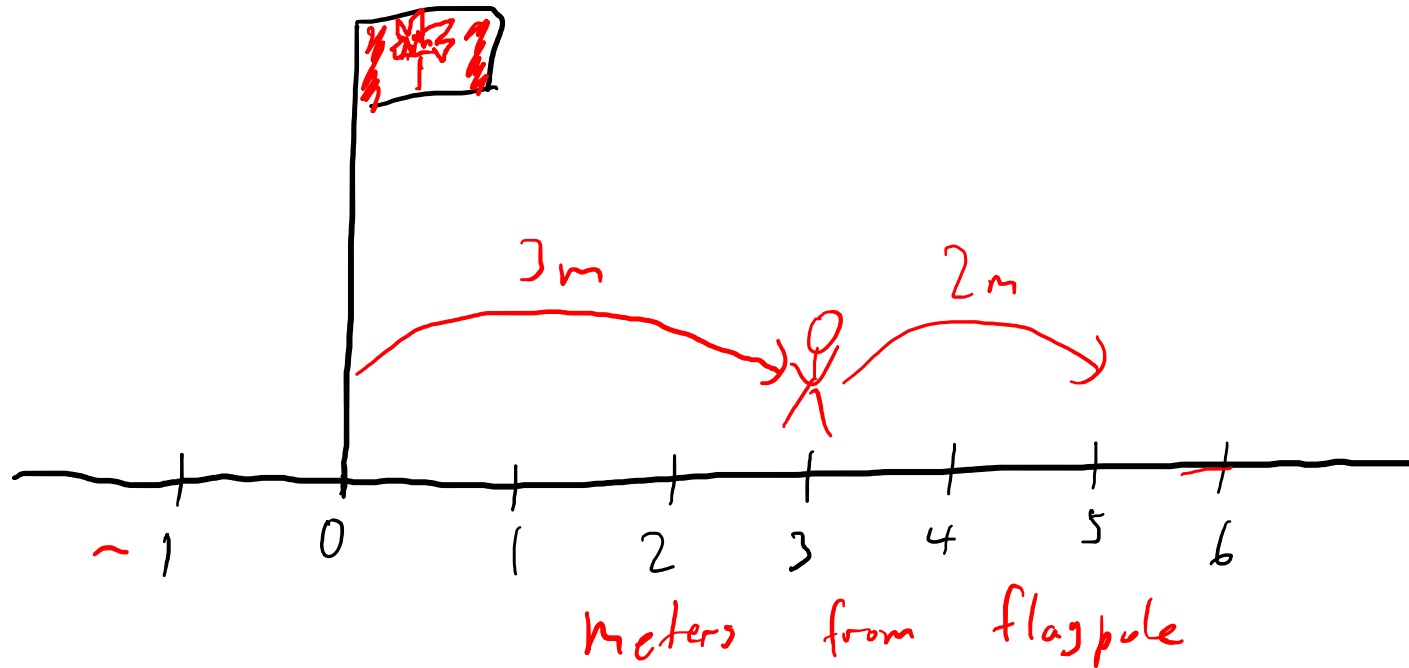
- Another way to understand addition of negative numbers is by moving to the left on the number line.

$$(-2) + (-3) = -5 \quad \left. \begin{array}{l} 3 + (-5) = \text{move right } 3 \\ \text{move left } 5 \end{array} \right\}$$

- Subtraction means to move in the opposite direction, or to add the negative of a number

Lengths and the positive number line

- One way positive numbers are used is to measure lengths



- Note however we only have whole number increments so far.

Think like a mathematician

- Mathematicians have a toolkit of problems they've solved already, and they try to turn a new problem into one they've seen before.
- Consider measuring the length of a rope by assigning a number on the positive number line.
- What lengths do we know how to directly compute so far?

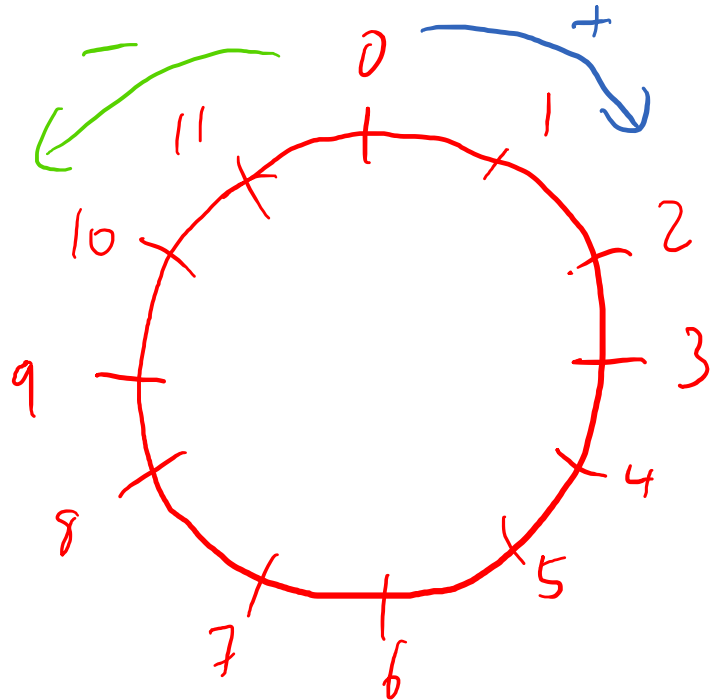


division
addition
multiplication
subtraction

- ~~A: Cutting the rope in half~~
- B: Joining together two ropes
- ~~C: Joining together 1000 ropes ?~~
- D: Cutting a piece of known length off the rope, and measuring the remainder.
- E: None of the above

Teaser for future lectures (week 6)

- Notice that we made a choice to invent negative numbers though. What if instead of making a copy of the numbers, we turn the number line into a number circle? This will be the basis for “clock arithmetic” or “modular arithmetic”.



$$0 - 1 = 11$$

$$11 + 2 = 1$$

