

The number line

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Think like a mathematician

- What problem remains after having invented negative numbers?

- A: We don't know how to subtract negative numbers
- B: We don't know how to add negative numbers
- C: We don't know how addition and subtraction interact
- D: All of the above
- E: None of the above

The positive number line

- We can arrange positive numbers on a line.
- The addition is moving to the right, subtraction to the left.

The negative number line

- The negative numbers are a copy of the positive numbers, so adding two negative numbers together should behave the same
- Similarly, subtracting two negative numbers should be similar

Combining the two number lines

- Can we combine the positive and negative number lines together? If so, how?

A: Stack on top of each other
B: Side-by-side
C: Side-by-side but flip one
D: One horizontal, one vertical
E: No, they cannot be combined

- What facts can we use to combine the two lines together?

A: Position of the 0
B: Definition of subtraction
C: The fact that they are copies
D: Extra dimensions
E: None of the above

The number line

- Remember, 0 is present in both lines, and subtraction of two positive numbers can give a negative number.
- Because of the way subtraction is defined, the negative copy of number line has to be flipped.

The number line

- Let's write the negative and positive numbers on a long line, with negative numbers to the left and positive numbers to the right:
- Another way to understand addition of positive numbers is how far to the right we are moving along the number line.
- Another way to understand addition of negative numbers is by moving to the left on the number line.
- Subtraction means to move in the opposite direction, or to add the negative of a number

Lengths and the positive number line

- One way positive numbers are used is to measure lengths
- Note however we only have whole number increments so far.

Think like a mathematician

- Mathematicians have a toolkit of problems they've solved already, and they try to turn a new problem into one they've seen before.
- Consider measuring the length of a rope by assigning a number on the positive number line.
- What lengths do we know how to directly compute so far?



- A: Cutting the rope in half
- B: Joining together two ropes
- C: Joining together 1000 ropes
- D: Cutting a piece of known length off the rope, and measuring the remainder.
- E: None of the above

Teaser for future lectures (week 6)

- Notice that we made a choice to invent negative numbers though. What if instead of making a copy of the numbers, we turn the number line into a number circle? This will be the basis for “clock arithmetic” or “modular arithmetic”.

