Inventing multiplication
Lecture 1c: 2022-01-12

MAT A02 – Winter 2022 – UTSC

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Repeated addition

- Say we want to count how many objects we have if we have four pairs of circles. Could simply draw them all out and count:

- Alternately, we're mathematicians, and we already invented addition, so we can add "2" repeatedly instead, and forget about the fact that they are circles.

$$\underbrace{2+2+2+2}_{4\ \text{times}} = 8$$

$$2+2+2+2$$
$$= 4+2+2$$
$$= 6+2 = 8$$

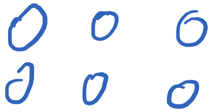
- This operation turns out to be very useful, so we name it "multiplication" and denote it by a times symbol 4×2 or $4 \cdot 2$

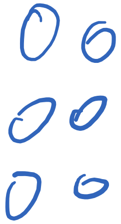
Multiplication table

- Like with addition/subtraction, we can write this into a table.

	0	1	2	3	4	...
0	0	0	0	0	0	
1	0	1	2	3	4	
2	0	2	4	6	8	
3	0	3	6	9	12	
4	0	4	8	12	16	
⋮						

$2+2+2=6$





$3+3=6$

When was multiplication invented?

- Remember that negative numbers were invented around 202 BCE-220 CE.

A: Before 1000 BCE
B: 1000 BCE to 1000 CE
C: 1000 CE to 1500 CE
D: 1500 CE to 1800 CE
E: After 1800 CE

(base 60)

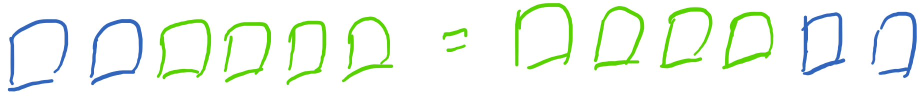
Circa 2000 BCE

~4000 years ago

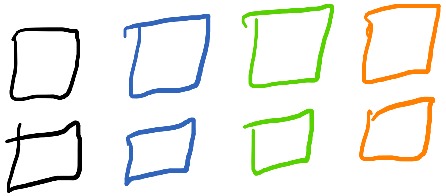
Babylonian 10-times table from Vorderasiatisches Museum, Berlin
http://www.ams.org/publicoutreach/feature-column/fc-2012-05

Properties

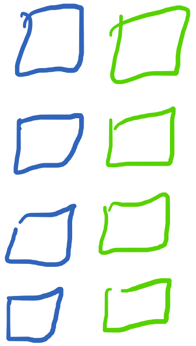
- Remember that $x + y = y + x$. What about $x \times y = y \times x$?



4 times 2:



2 times 4:



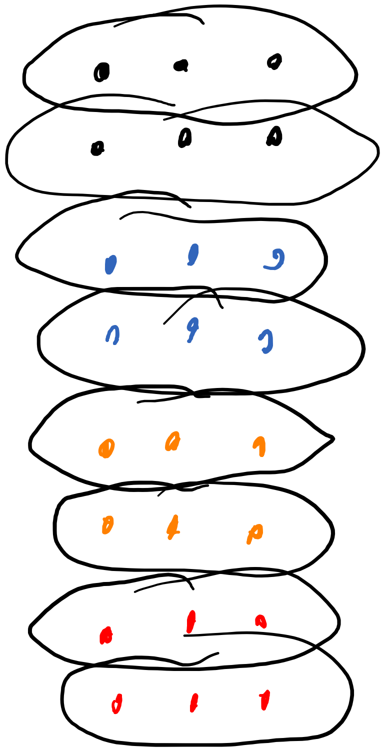
just turn sideways

A: Commutative
B: Associative
C: Identity
D: Distributive
E: None of the above

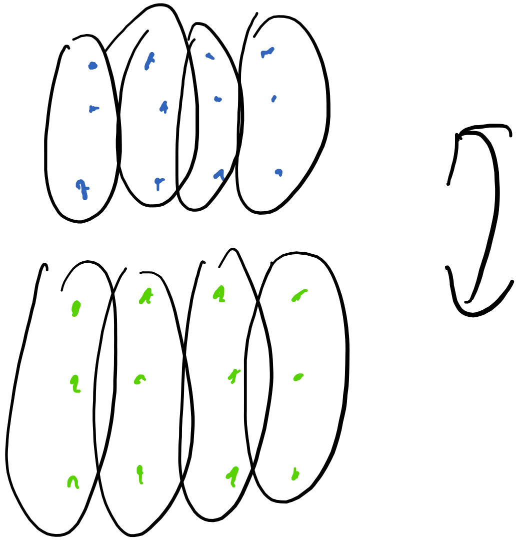
Properties

- $(x \times y) \times z = x \times (y \times z)$

$(2 \times 3) \times 4$



$2 \times (3 \times 4)$



$= 8 \times 3 = 24$

A: Commutative
B: Associative
C: Identity
D: Distributive
E: None of the above

What happens to negative numbers?

- Let's write out the a multiplication table and continue the pattern:

	-3	-2	-1	0	1	2	3
-3	9	6	3	0	-3	-6	-9
-2	6	4	2	0	-2	-4	-6
-1	3	2	1	0	-1	-2	-3
0	0	0	0	0	0	0	0
1	-3	-2	-1	0	1	2	3
2	-6	-4	-2	0	2	4	6
3	-9	-6	-3	0	3	6	9

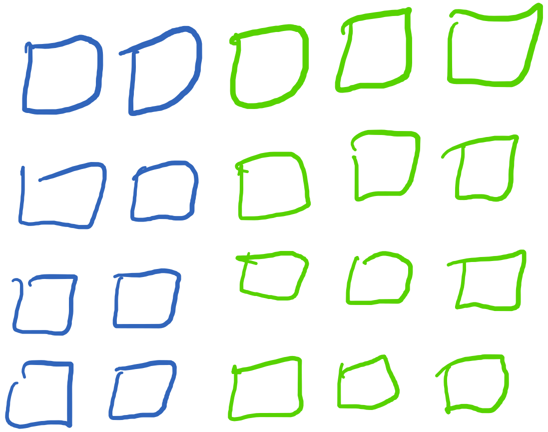
neg × pos = neg

neg × neg = pos

Combining addition and multiplication

- What happens if you add two numbers together and then multiply?

$$4 \times (2+3) = 4 \times 2 + 4 \times 3$$



because we can count the number of 4's we add

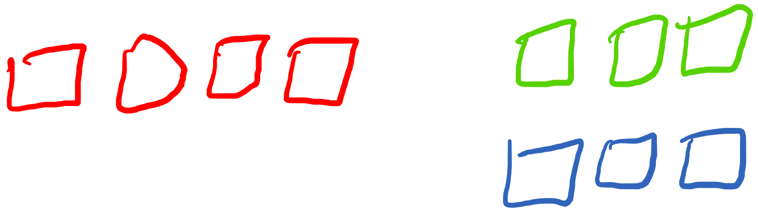
- This gives rise to the distributive property:

$$x \times (y+z) = x \times y + x \times z$$

Caution!

- What happens if you multiply first, and then add?

$$4 + (2 \times 3) = 10$$



- There is no comparable rule to the distributive property. You just have to do the multiplication, and then do the addition.

Addition and multiplication properties

1 + 5 = 5 + 1
2 · 3 = 3 · 2

- Commutative property:

$$x + y = y + x$$
$$x \times y = y \times x$$

Swapping parens

- Associative property:

$$(x + y) + z = x + (y + z)$$
$$(x \times y) \times z = x \times (y \times z)$$

- Identity properties:

$$0 + x = x$$
$$1 \times x = x$$

- Distributive property:

$$x \times (y + z) = xy + xz$$

Try it out: solving a linear equation

$a+b=b+a$

$-5-x=-x-5$

- $5(x+2)-1=(x+1)-5-x+18$

$5(x+2)-1=(x+1)-x-5+18$ commutative

$5(x+2)-1=x+(1-x)-5+18$ assoc.

$5(x+2)-1=x+(-x+1)-5+18$ commut

$5(x+2)-1=(x-x)+1-5+18$ assoc

$5(x+2)-1=0+1-5+18$ none of the above

$5(x+2)-1=1-5+18$ identity $0+1=1$

$(5x+10)-1=1-5+18$ distributive

$5x+(10-1)=1-5+18$ associative

$5x+9=14$ none of the above

$5x=5$ ← stuck   subtracted 9 from both sides

A: Commutative
B: Associative
C: Identity
D: Distributive
E: None of the above

$(a+b)+c=a+(b+c)$