Inventing multiplication
Lecture 1c: 2022-01-12

MAT A02 – Winter 2022 – UTSC

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Repeated addition

- Say we want to count how many objects we have if we have four pairs of circles. Could simply draw them all out and count:
- Alternately, we're mathematicians, and we already invented addition, so we can add "2" repeatedly instead, and forget about the fact that they are circles.
- This operation turns out to be very useful, so we name it "multiplication" and denote it by a times symbol 4×2 or $4 \cdot 2$

Multiplication table

- Like with addition/subtraction, we can write this into a table.

When was multiplication invented?

- Remember that negative numbers were invented around 202 BCE-220 CE.

A: Before 1000 BCE
B: 1000 BCE to 1000 CE
C: 1000 CE to 1500 CE
D: 1500 CE to 1800 CE
E: After 1800 CE

Babylonian 10-times table from Vorderasiatisches Museum, Berlin
http://www.ams.org/publicoutreach/feature-column/fc-2012-05

Properties

- Remember that $x + y = y + x$. What about $x \times y = y \times x$?

A: Commutative
B: Associative
C: Identity
D: Distributive
E: None of the above

Properties

- $(x \times y) \times z = x \times (y \times z)$

A: Commutative
B: Associative
C: Identity
D: Distributive
E: None of the above

What happens to negative numbers?

- Let’s write out the a multiplication table and continue the pattern:

Combining addition and multiplication

- What happens if you add two numbers together and then multiply?

- This gives rise to the distributive property:

$$x \times (y + z) = x \times y + x \times z$$

Caution!

- What happens if you multiply first, and then add?

- There is no comparable rule to the distributive property. You just have to do the multiplication, and then do the addition.

Addition and multiplication properties

- Commutative property:

$$x + y = y + x$$
$$x \times y = y \times x$$

- Associative property:

$$(x + y) + z = x + (y + z)$$
$$(x \times y) \times z = x \times (y \times z)$$

- Identity properties:

$$0 + x = x$$
$$1 \times x = x$$

- Distributive property:

$$x \times (y + z) = xy + xz$$

Try it out: solving a linear equation

- $5(x+2)-1=(x+1)-5-x+18$

A: Commutative
B: Associative
C: Identity
D: Distributive
E: None of the above