Combinations
Lecture 3a: 2022-01-24

MAT A02 – Winter 2022 – UTSC

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Let’s bake a cake

- We’re going to have to measure ingredients.

Can we measure the right amount?

- Let's say we want to measure 5 tsp cinnamon.
- Can we do it with a 2 tsp spoon and a 1tbsp = 3 tsp spoon?

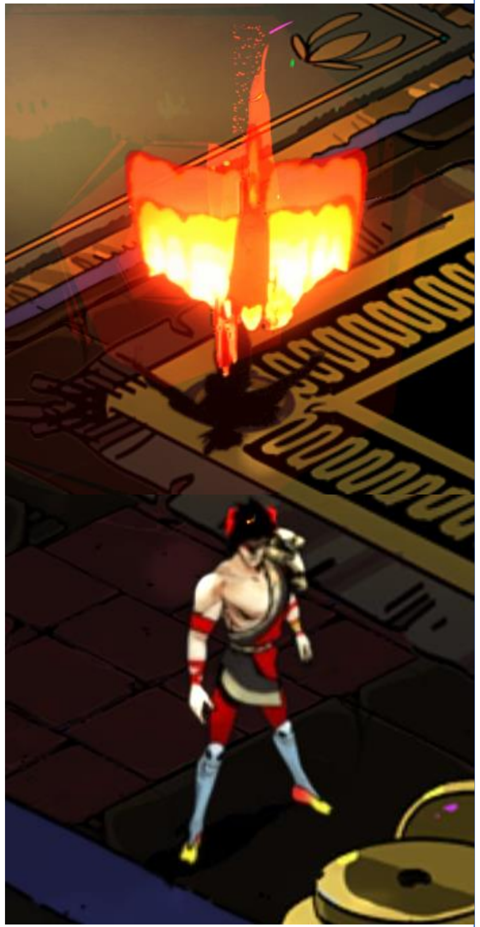
- What about 1tbsp = 3 tsp spoon and a third cup = 16 tsp cup?

- What about a 4 tsp spoon and a 1 cup = 48 tsp cup?

A: Yes
B: No
C: Yes, but only in theory
E: None of the above

Let’s play a game

- In some video games, can move normally, or can do a fixed-length “dash”.
- When playing with a keyboard, a single button press may also go a specific distance.
- If the dash goes 9 ft, and walking goes 2 ft, can you go exactly 25 ft?



- What if you speed up walking to go 3 ft?

A: Yes
B: No
C: Yes, but only in theory
E: None of the above

Combinations

- Given two integers a and b, the sum of a multiple of a and a multiple of b (allowing negative multiples) is called a combination of a and b.

  $a \cdot x + b \cdot y$, where x, y are integers.

- Given two integers a and b, what numbers n are combinations of a and b?
- If n is a combination of a and b, how do we find multiples of a and b actually adding up to n?

Creating Unity

- Once you’ve figured out how to go 1 step, you could in theory figure out how to go any whole number of steps just by repeating the whole procedure.
- Example: 1tbsp = 3 tsp spoon and a third cup = 16 tsp cup
- Then we just repeat that any number of times, showing that we can make every number as a combination.

Process for making smaller combos?

- Let's keep on making smaller combinations, and then use those as building blocks.
- Ex. Combinations of 35 and 100.
- Can repeatedly subtract 35 from 100 to get a smaller combo.
- Can rewrite as division with remainder.
- Now we know how to make 30, so can use as a building block.
- But now we're stuck at 5, since $30 \div 5 = 6$.

The Euclidean algorithm and combinations

- If we start with two whole numbers a and b, then the smallest combo we can make uses the following algorithm:
- Divide a into b, and let r be the remainder.
  - If $r = 0$, then we're done; a divides b and $\gcd(a, b) = a$.
  - If $r \neq 0$, then we replace (a, b) with (r, a) and repeat.
- This is just the Euclidean algorithm, so the smallest combo we can make is precisely the greatest common divisor $\gcd(a, b)$.
- This means we know how to make any multiple of the $\gcd(a, b)$.

Other combinations?

- Can we make any positive numbers that are not multiples of $\gcd(a, b)$?

A: Yes
B: No
C: Yes, but only in theory
E: None of the above

- The set of combos of a, b is precisely the multiples of $\gcd(a, b)$

Try it out

- What is the set of combinations of 36 and 3222?

A: Multiples of 9
B: Multiples of 18
C: Multiples of 36
D: Multiples of 72
E: None of the above

Try it out (recall)

- Find the least common multiple of 36 and 3222?

A: 3222
B: 6444
C: 9333
D: 12888
E: None of the above

Try it out

- Find integers x, y such that $36x + 3222y = 9$

A: $x = 178, y = 2$
B: $x = 89, y = 1$
C: $x = -89, y = 1$
D: $x = -178, y = 2$
E: None of the above

Try it out

- Find integers x, y such that $36x + 3222y = 18$

A: $x = 178, y = 2$
B: $x = 89, y = 1$
C: $x = -89, y = 1$
D: $x = -178, y = 2$
E: None of the above

Try it out

- Find integers x, y such that $36x + 3222y = 36$

A: $x = 178, y = 2$
B: $x = 89, y = 1$
C: $x = -89, y = 1$
D: $x = -178, y = 2$
E: None of the above

Try it out

- Find integers x, y such that $36x + 3222y = 45$

A: $x = 178, y = 2$
B: $x = 89, y = 1$
C: $x = -89, y = 1$
D: $x = -178, y = 2$
E: None of the above

More complicated example

More complicated example