

# 2022-01-26-quiz-review

Wednesday, January 26, 2022 10:36 AM

$$\frac{2}{3}x + \frac{8}{5} = \frac{3}{2}x + \frac{7}{20}$$

$$\underline{x20}: \frac{40}{3}x + 32 = 30x + 7$$

$$\underline{x3}: 40x + 96 = 90x + 21$$

$$50x = 75$$

$$x = \frac{75}{50} = \frac{3}{2}$$

$$\underline{x60}: 40x + 96 = 90x + 21$$

$$50x = 75$$

$$x = \frac{3}{2} = 1\frac{1}{2}$$

$$\text{gcd}(63, 12)$$

$$= \text{gcd}(51, 12)$$

$$= \text{gcd}(39, 12)$$

$$= \text{gcd}(27, 12)$$

$$= \text{gcd}(15, 12)$$

$$= \text{gcd}(3, 12)$$

Example: 75, 1938  $d = \text{gcd}(75, 1938)$

Euclidean algorithm

$$\begin{array}{r} 25 \text{ r } 63 \\ 75 \overline{)1938} \\ \underline{150} \\ 438 \\ \underline{375} \\ 63 \end{array}$$

$$\rightarrow 1938 = 75 \cdot 25 + 63 \Rightarrow d = \text{gcd}(75, 63)$$

$$\rightarrow 75 = 63 \cdot 1 + 12 \Rightarrow d = \text{gcd}(63, 12)$$

$$\rightarrow 63 = 12 \cdot 5 + 3 \Rightarrow d = \text{gcd}(12, 3)$$

$$12 = 3 \cdot 4 \Rightarrow \underline{\underline{d = 3}}$$

$$\begin{array}{r} 1 \text{ r } 12 \\ 63 \overline{)75} \\ \underline{63} \\ 12 \end{array}$$

Want 3:  $75 \cdot x + 1938 \cdot y$ ,  $x, y$  integers

$$\rightarrow 3 = 63 - 12 \cdot 5$$

$$\rightarrow 3 = 63 - (75 - 63) \cdot 5$$

$$3 = 63 - 75 \cdot 5 + 63 \cdot 5$$

$$\bullet 3 = 63 \cdot 6 - 75 \cdot 5$$

$$\bullet 3 = (1938 - 75 \cdot 25) \cdot 6 - 75 \cdot 5$$

$$\bullet 3 = 1938 \cdot 6 - 75 \cdot 25 \cdot 6 - 75 \cdot 5$$

$$\bullet 3 = 1938 \cdot 6 - 75(25 \cdot 6 + 5)$$

$$\bullet 3 = 1938 \cdot 6 - 75(150 + 5)$$

$$\boxed{3 = 1938 \cdot 6 - 75 \cdot 155}$$

$$75 = 63 + 12$$

$$12 = 75 - 63$$

$$1938 = 75 \cdot 25 + 63$$

$$63 = 1938 - 75 \cdot 25$$

$$3 = 63 - (75 - 63) \cdot 5$$

$$3 = 63 - [75 \cdot 5 - 63 \cdot 5]$$

$$3 = 63 - 75 \cdot 5 + 63 \cdot 5$$

$$3 = 63 + 63 \cdot 5 - 75 \cdot 5$$

$$3 = 63(1 + 5) - 75 \cdot 5$$

$$\hookrightarrow 3 = 63 \cdot 6 - 75 \cdot 5$$

$$3 = 1938 \cdot 6 - 75 \cdot 155$$

$$3 = 63(175) \\ 3 = 63 \cdot 6 - 75 \cdot 5$$

$$1938 = 75 \cdot 25 + 63$$

$$1938 \div 75 = 25 \text{ remainder } 63$$

$$\frac{1938}{75} = 25 \frac{63}{75} = 25 + \frac{63}{75}$$

$$1938 = 75 \cdot 25 + 63$$

$$3 = 63 - 12 \cdot 5$$

$$3 = 63 - a \cdot 5$$

$$3 = 63 - (75 - 63) \cdot 5$$

$$75 = 63 + 12$$

$$12 = 75 - 63$$

$$a = 12$$

$$a = 75 - 63$$

Fact:  $a \cdot b = \text{gcd}(a, b) \cdot \text{lcm}(a, b)$

$$\text{lcm}(a, b) = \frac{a \cdot b}{\text{gcd}(a, b)}$$

$$\text{lcm}(1938, 75) = \frac{1938 \cdot 75}{3} = 1938 \cdot 25 \\ = 2,969 \cdot 25$$

$$1938 = 2 \cdot 969$$

$$\begin{array}{r} 484 \text{ r } 1 \\ 2 \overline{) 969} \end{array}$$

$$= 969 \cdot 50$$

$$= \frac{969}{2} \cdot 100$$

$$= 484.5 \cdot 100$$

$$= 48,450$$



$$d = \gcd(6, 9)$$

$$9 \div 6 = 1 \text{ r } 3$$

$$6 \div 3 = 2$$

$$3 = 9x + 6y$$

$$9 = 6 \cdot 1 + 3$$



$$3 = 9 - 6$$

$$d = \gcd(6, 3)$$

$$d = 3$$

$$d = \gcd(11, 43)$$

$$43 \div 11 = 1 \text{ r } 10$$

$$11 \div 10 = 1 \text{ r } 1$$

$$10 \div 1 = 10 \text{ r } 0 \quad \checkmark$$

$$1 = 11x + 10y$$

$$43 = 11 \cdot 3 + 10$$

$$11 = 10 \cdot 1 + 1$$

$$1 = 11 - 10$$

$$1 = 11 - (43 - 11 \cdot 3)$$

$$1 = 11 - 43 + 11 \cdot 3$$

$$1 = 11 \cdot (1 + 3) - 43$$

$$1 = 11 \cdot 4 - 43$$

$$d = \gcd(11, 10)$$

$$d = \gcd(10, 1)$$

$$d = 1$$

$$43 = 11 \cdot 3 + 10$$

$$10 = 43 - 11 \cdot 3$$

2. Rewrite exponents.

$$\left( \frac{x^2 y^4 z^{-8} z^0 z^2}{16 x^{-4} y^0 z^1} \right)^{-\frac{1}{2}} = \left( \frac{x^{2-(-4)} y^{4-8+0} z^{2-1}}{16} \right)^{-\frac{1}{2}}$$

$$= \left( \frac{x^6 y^{-4} z}{16} \right)^{-\frac{1}{2}}$$

$$= \left( \frac{16}{x^6 y^{-4} z} \right)^{\frac{1}{2}} = \left( \frac{16 y^4}{x^6 z} \right)^{\frac{1}{2}}$$

$$= \left( \frac{x^{-6} y^4 z^{-1}}{16^{-1}} \right)^{\frac{1}{2}} = \left( \frac{16 y^4}{x^6 z} \right)^{\frac{1}{2}}$$

*Handwritten notes:  $\frac{1}{2}$  (yellow),  $16 \sqrt{\quad}$ ,  $4 \dots 2$*

$$= \frac{16y}{x^{6 \cdot \frac{1}{2}} z^{1 \cdot \frac{1}{2}}} = \frac{4y^2}{x^3 z^{\frac{1}{2}}}$$

$$= \left( \frac{4y^2}{x^6 z} \right)^{\frac{1}{2}}$$

$$\left( \frac{x^2 y^4 z^{-8} z^0 z^2}{16 x^{-4} y^0 z} \right)^{-\frac{1}{2}}$$

$$= \frac{x^{-1} y^{-2} y^4 z^0 z^{-1}}{16^{-\frac{1}{2}} x^2 y^0 z^{-\frac{1}{2}}} = \frac{16^{\frac{1}{2}} y^2}{x^3 z^{\frac{1}{2}}} = \frac{4y^2}{x^3 z^{\frac{1}{2}}}$$