LCM and GCD via factorization
Lecture 5a: 2022-02-07

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Greatest common divisor

- Suppose d divides n. Then prime factorization of d must have exponents less than or equal to prime factorization of n.
- If d divides both n and m, then all of the exponents must be less than or equal to those in both n and m.
- The greatest common divisor thus has exponents exactly equal to the smaller of the exponents in n and m.

Least common multiple

- Suppose m is a multiple of a. Then prime factorization of m must have exponents greater than or equal to prime factorization of a.
- If m is a multiple of both a and b, then all of the exponents must be greater than or equal to those in both a and b.
- The least common multiple thus has exponents exactly equal to the larger of the exponents in n and m.

$$\mathrm{lcm}(a,b) \cdot \gcd(a,b) = ab$$

- The lcm gets all the smaller exponents, and the gcd gets all the larger exponents, so multiplied together, you get all of the original exponents in a and b.

Try it out

- What is the greatest common divisor of 3072 and 896?
- Step 1: what is the prime factorization of 3072?

A: $2^8 \cdot 5$
B: $2^7 \cdot 7$
C: $2^{10} \cdot 3$
D: $2^5 \cdot 3^5$
E: None of the above

- Step 2: what is the prime factorization of 896?

A: $2^5 = 32$
B: $2^7 = 128$
C: $2^7 \cdot 3 \cdot 5 = 1920$
D: $2^{10} \cdot 3 \cdot 7 = 21504$
E: None of the above

- Step 3: get the smallest of each exponent.