Review session: combos and primes Lecture 5d: 2022-02-09

MAT A02 – Winter 2022 – UTSC

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Prime factorization

- For any positive integer n, can factor by attempting to divide by primes. Only have to check up to $\sqrt{n}$.

Try it out

- What is the prime factorization of 12240?

A: $2^4 \cdot 3^2 \cdot 5 \cdot 17$
B: $2^3 \cdot 3^2 \cdot 5^2 \cdot 7^2$
C: $2^3 \cdot 3^3 \cdot 5 \cdot 13$
D: $2^4 \cdot 7 \cdot 17^2$
E: None of the above

Greatest common divisor

- Use either Euclidean algorithm or prime factorization.

A: 10
B: 20
C: 25
D: 50
E: None of the above

Advanced Combinations

- Given two integers, m and n, can solve for the combination $\gcd(m, n) = mx + ny$ by reversing the Euclidean algorithm.
- Can solve for any multiple $c \cdot \gcd(m, n) = mx + ny$ by multiplying the above solution by c.
- Can solve for all combinations $0 = mx + ny$ by dividing by all common factors of m and n and considering all multiples of $(x = m, y = -n)$
- Can add 0 to any other solution to get different combinations.

Try it out

- Is there an integer combination of 100 and 12240 that is equal to 50?
- What about 60?
- Can you find three different solutions?

A: Yes
B: No

More solutions

Counting divisors

- Take all the exponents in the prime factorization of n, add 1 to each of them, and then take the product.

Try it out

- How many divisors does 100 have?

A: 6
B: 8
C: 9
D: 10
E: None of the above

Counting common divisors

- Just need to find the divisors of the greatest common divisor.
- How many numbers are divisors of both 100 and 12240?

A: 6
B: 8
C: 9
D: 10
E: None of the above

Euler's ϕ function

- Can use modified sieve of Eratosthenes to remove all numbers that are not relative primes.
- Alternately, each time we remove multiples of a prime p, we remove $\frac{1}{p}$ of the remaining numbers.
- So if a prime factorization is $n = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}$, where each $a_i > 0$, then

$$\phi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\cdots\left(1 - \frac{1}{p_k}\right)$$

Try it out

- $\phi(1000)$

- $\phi(3993)$

A: 100
B: 400
C: 1210
D: 2420
E: None of the above