Pi Day Lecture: figuring out pi
Lecture 9b: 2022-03-14

MAT A02 – Winter 2022 – UTSC

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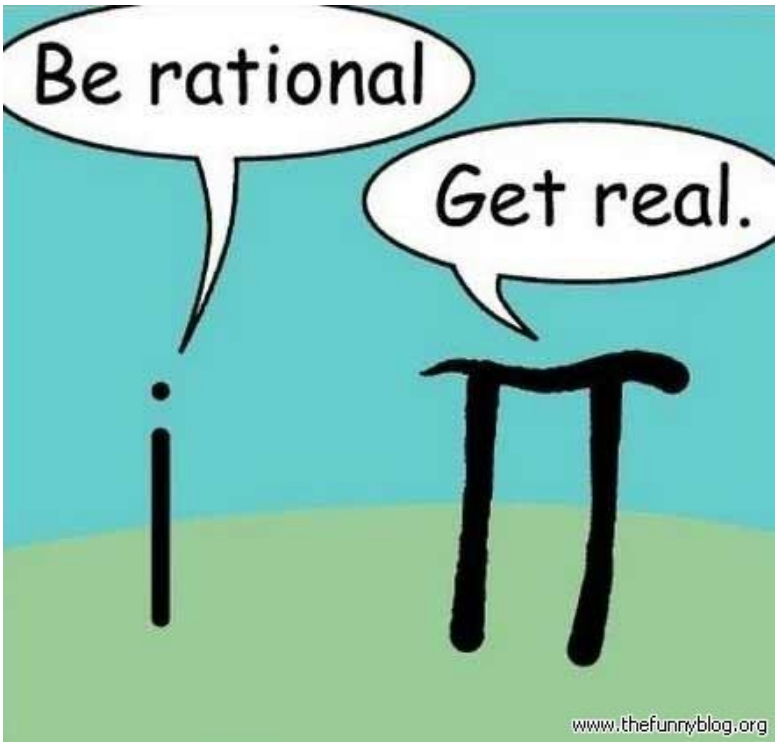
What is pi?

3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808651328230664709384460955058223172535940812848111745028410270193852110555964462294895493038196442881097566593344612847564823378678316527120190914564856692346034861045432664821339360726024914127372458700660631558817488152092096282925409171536436789259036001133053054882046652138414695194151160943305727036575959195309218611738193261179310511854807446237996274956735188575272489122793818301194912983367336244065664308602139494639522473719070217986094370277053921717629317675238467481846766940513200056812714526356082778577134275778960917363717872146844090122495343014654958537105079227968925892354201995611212902196086403441815981362977477130996051870721134999999837297804995105973173281609631859502445945534690830264252230825334468503526193118817101000313783875288658753320838142061717766914730359825349042875546873115956286388235378759375195778185778053217122680661300192787661119590921642019892.....



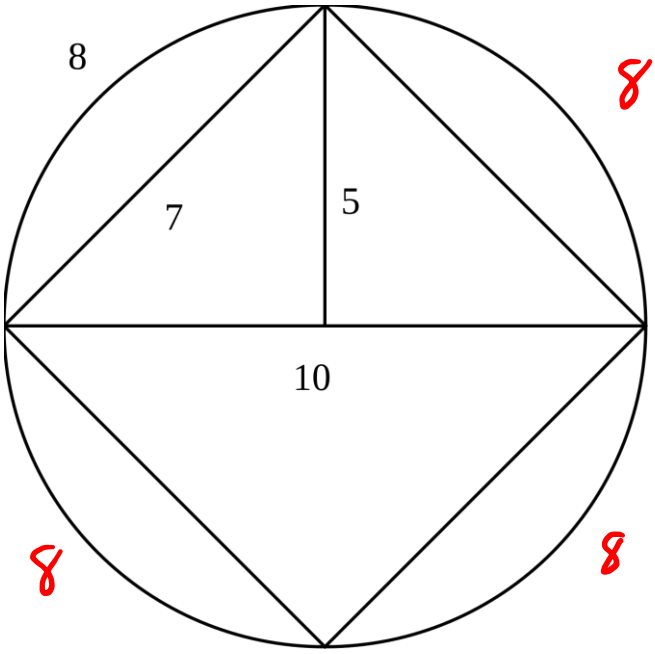
Hebrew Bible

- Hebrew Bible: $\pi \approx 3$ (see Solomon's Temple)

1 Kings 7:23. And he made the Sea of cast bronze, ten cubits from one brim to the other; it was completely round. Its height was five cubits, and a line of thirty cubits measured its circumference

Approximating Pi

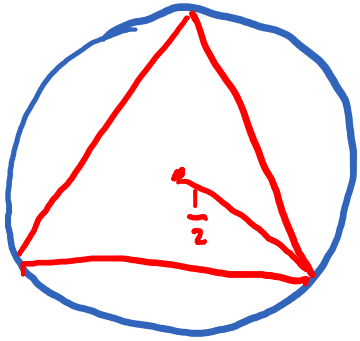
Indiana Pi bill, 1897

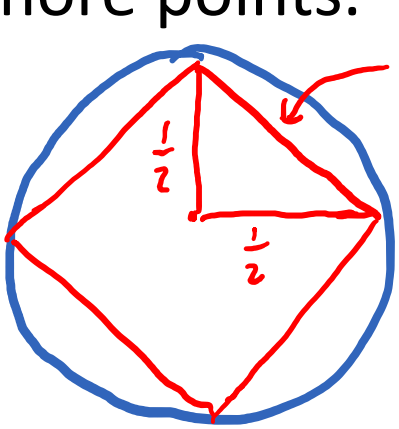


$\pi \approx 3.2$

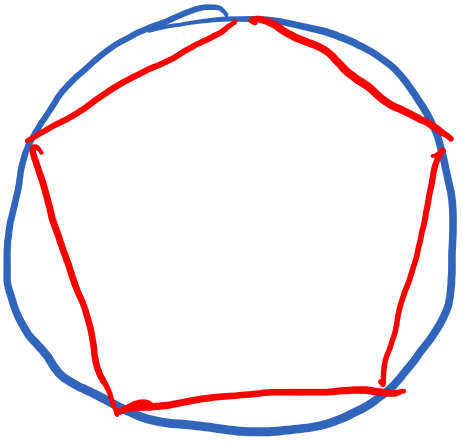
Successive polygons

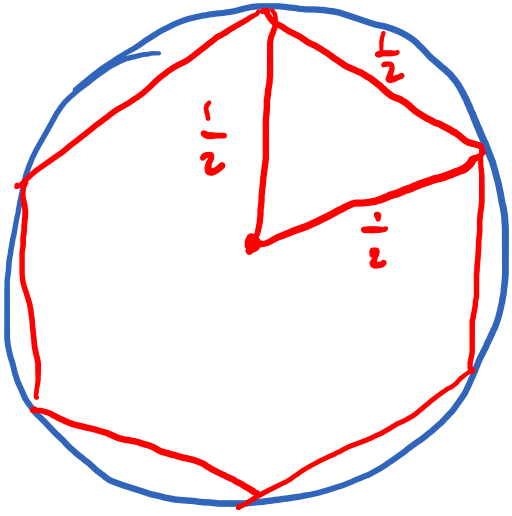
- Use polygons with more points.





$\pi \approx 2\sqrt{2} \approx 2.828$





$\pi \approx 3$

Historical approximations

- 1550 BCE (Egypt, Rhind mathematical papyrus):
  - $\pi \approx \frac{256}{81} \approx 3.16$ (using octagon)
- 600 BCE (India, Shatapatha Brahmana):
  - $\pi \approx \frac{339}{108} \approx 3.139$ (for astronomy)
- 300 BCE (Archimedes, Greece):
  - $\frac{223}{71} < \pi < \frac{22}{7}$ (regular 96-gons)
- 200 BCE (Ptolemy, Greece):
  - $\pi \approx \frac{377}{120} \approx 3.141666$
- 263 CE (Liu Hui, China):
  - $3.141024 < \pi < 3.142708$ (96-gon and 192-gon)
- 600 CE (Aryabhata, India):
  - $\pi \approx \frac{62832}{20000} = 3.1416.$

Complicated algebraic formulas

- 14th century Indian mathematician Madhava of Sangamagrama:

$$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \ldots\right)$$

- 1593: French mathematician Francois Viete:

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \ldots$$

- 1910: Srinivasa Ramujan (Indian, worked in Cambridge, UK):

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!\,(1103 + 26390k)}{(k!)^4 396^{4k}}$$

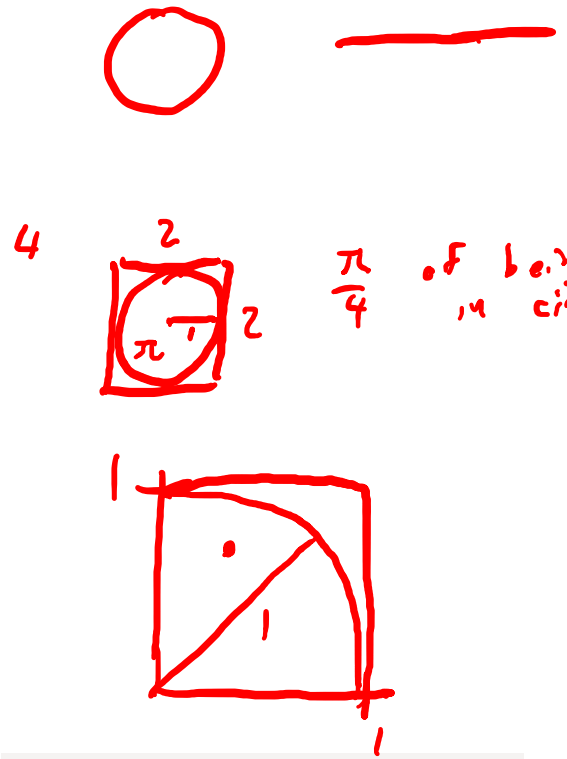
Empirical measurement of pi

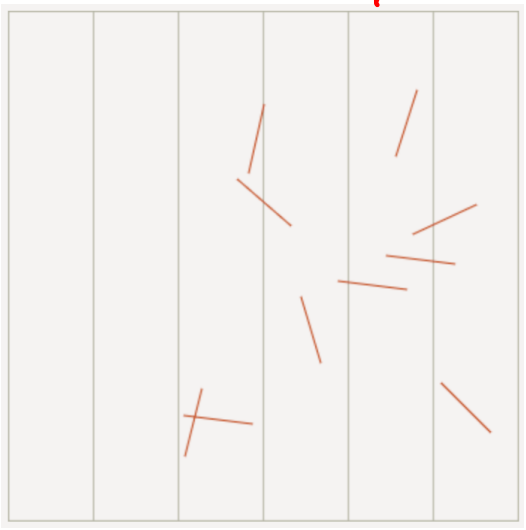
- Take a rope and wrap it around something circular, and just measure the ratio of the circumference to the diameter.
- Roll dice to find n random points in the unit square. See how many of them (m) are within distance 1 of the origin using the Pythagorean theorem.

$$\pi \approx \frac{4m}{n}$$

- Drop n toothpicks length t between evenly spaced lines with spacing l, and count how many sticks m cross a line.

$$\pi \approx \frac{2ln}{mt}$$





https://www.sciencefriday.com/articles/estimate-pi-by-dropping-sticks/