# Pi Day Lecture: figuring out pi Lecture 9b: 2022-03-14

MAT A02 – Winter 2022 – UTSC

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# What is pi?

3.1415926535897932384626433832795028841971
693993751058209749445923078164062862089986
280348253421170679821480865132823066470938
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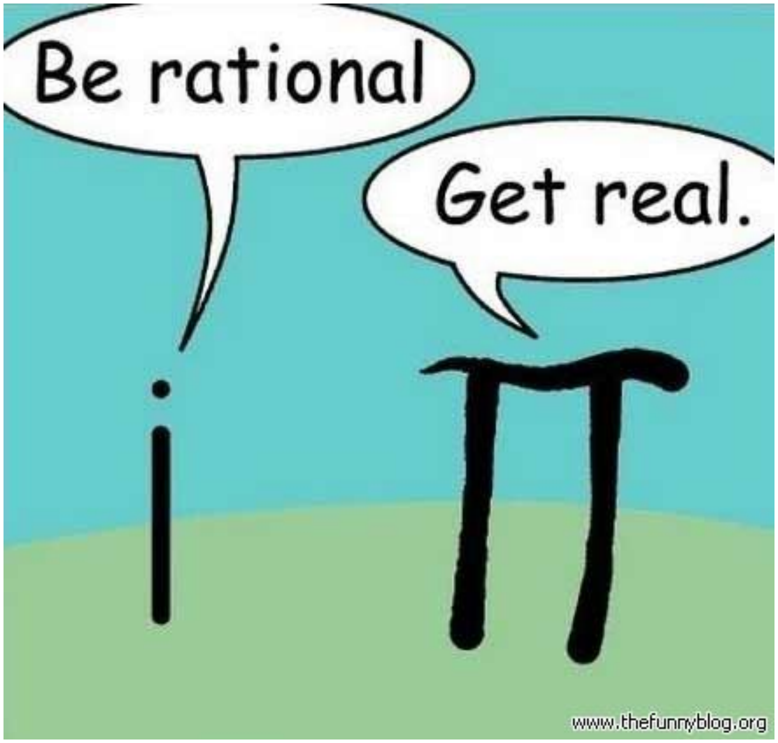
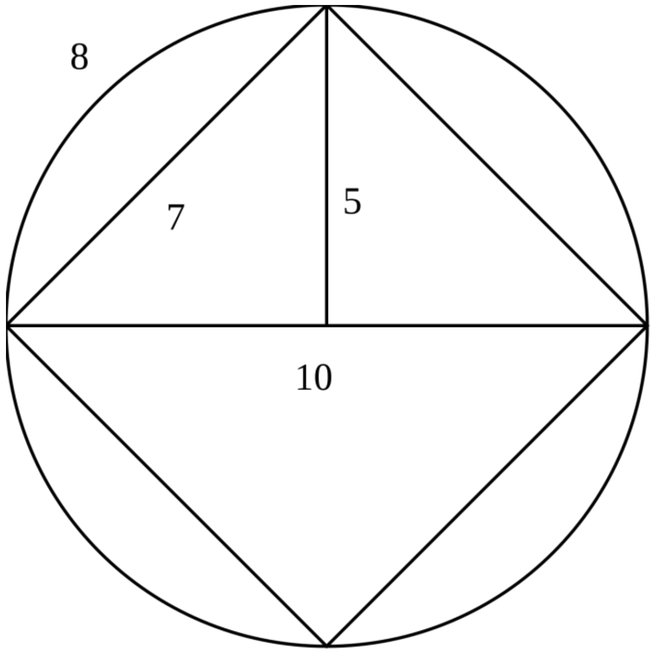


# Hebrew Bible

- Hebrew Bible: $\pi \approx 3$ (see Solomon's Temple)

1 Kings 7:23. And he made the Sea of cast bronze, ten cubits from one brim to the other; it was completely round. Its height was five cubits, and a line of thirty cubits measured its circumference

# Approximating Pi

# Successive polygons

- Use polygons with more points.

# Historical approximations

- 1550 BCE (Egypt, Rhind mathematical papyrus):
  - $\pi \approx \frac{256}{81} \approx 3.16$ (using octagon)
- 600 BCE (India, Shatapatha Brahmana):
  - $\pi \approx \frac{339}{108} \approx 3.139$ (for astronomy)
- 300 BCE (Archimedes, Greece):
  - $\frac{223}{71} < \pi < \frac{22}{7}$ (regular 96-gons)
- 200 BCE (Ptolemy, Greece):
  - $\pi \approx \frac{377}{120} \approx 3.141666$
- 263 CE (Liu Hui, China):
  - $3.141024 < \pi < 3.142708$ (96-gon and 192-gon)
- 600 CE (Aryabhata, India):
  - $\pi \approx \frac{62832}{20000} = 3.1416.$

# Complicated algebraic formulas

- 14th century Indian mathematician Madhava of Sangamagrama:

$$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \dots\right)$$

- 1593: French mathematician Francois Viete:

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \dots$$

- 1910: Srinivasa Ramujan (Indian, worked in Cambridge, UK):

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!\,(1103 + 26390k)}{(k!)^4 396^{4k}}$$

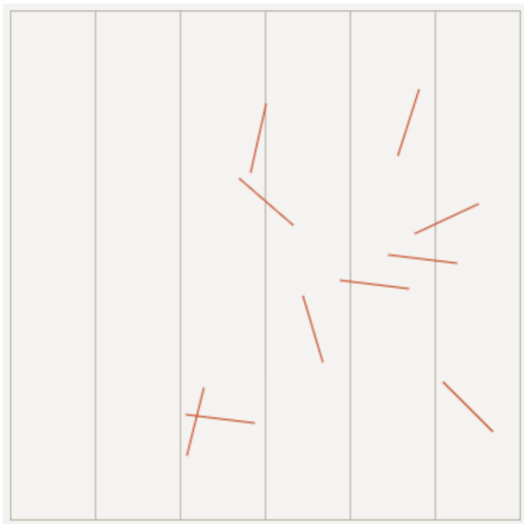
# Empirical measurement of pi

- Take a rope and wrap it around something circular, and just measure the ratio of the circumference to the diameter.
- Roll dice to find $n$ random points in the unit square. See how many of them ($m$) are within distance 1 of the origin using the Pythagorean theorem.

$$\pi \approx \frac{m}{n}$$

- Drop $n$ toothpicks length $t$ between evenly spaced lines with spacing $l$, and count how many sticks $m$ cross a line.

$$\pi \approx \frac{2ln}{mt}$$



https://www.sciencefriday.com/articles/estimate-pi-by-dropping-sticks/