



MATA02H3 Winter 2022 Midterm - Practice Student ID: _____
Computer and Mathematical Sciences University of Toronto at Scarborough

First name (please write as legibly as possible within the boxes)

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Last name

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Student ID number

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- Write your Student ID at the top and in the box.
- This exam has 7 questions and 9 pages.
- In order to receive credit, you must show your work.
- Simplify your answers whenever possible.
- A calculator (with no memory storage) is allowed.
- Phones and other electronic devices are not permitted.

Question	Points	Score
1	15	
2	16	
3	12	
4	15	
5	15	
6	15	
7	12	
Total	100	

First 25 primes: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



1. Solve linear equations. Show your work.

(a) $\frac{2}{7}x - 2 = \frac{1}{2}x - \frac{23}{10}$

(b) $\frac{2}{5}x - \frac{1}{3} = \frac{1}{2}x - \frac{7}{10}$

(c) $\frac{1}{2}(x + 5) = \frac{7}{5}x - \frac{3}{10}$



2. Simplify the following expressions.

- (a) Assume that all variables represent positive real numbers, and write each of the following as a product or quotient of powers in which each variable appears only once.

$$\left(\frac{z^4 x^1 x^{-3} z^2}{16x^4 y^2 z^3} \right)^{-\frac{1}{2}}$$

- (b) Rewrite the following in the form $\frac{p}{q}$ where p and q are integers, and the fraction is as simplified as possible. Do not write your answer as a mixed number.

$$\frac{4^2 \cdot 9}{2^3 \cdot 11} + \left(\frac{48}{4 + 2} \right)^{\frac{1}{3}}$$



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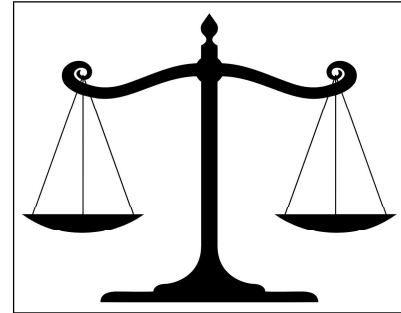
3. What are the greatest common divisor and least common multiple of each of the following pairs of numbers 2310 and 3600?

(a) (2310, 3600)

(b) (843, 2397)



4. Suppose that you have an old-school balance scale, which tells you if the two sides are the same weight. Unfortunately, you lost your calibrated weights. However, you do still have a set of spoons, each of which you know weighs 26 grams. You also have a set of bowls, each of which you know weighs 100 grams.



(a) What measurements are you able to do? i.e. what weight objects can you precisely measure by balancing spoons and bowls on either side?

(b) How would you measure out the weight of an object that is 24 grams? How many spoons and bowls do you need to have?

(c) Can you figure out a way that uses fewer spoons and bowls?



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5. **Practice:** Factorization, divisors, and relative primes.

(a) What is the prime factorization of 371498400?

(b) How many divisors does 371498400 have? Write out 10 of the divisors.

(c) How many positive integers less than 371498400 are relatively prime to 371498400?



6. Compute the following in modular arithmetic

(a) $2832412 \times 512 \pmod{64}$

(b) $2945234 + 1497463 \pmod{45}$

(c) $111^2 \pmod{250}$

(d) $5^{64} \pmod{23}$

(e) Find all the powers of 3 in modulo 19.



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7. In class, we've seen the invention of positive integers (1,2,3,...), negative integers (-1, -2, -3,...), rational numbers ($\frac{1}{2}$, $\frac{21}{56}$, ...), irrational numbers ($\sqrt{2}$, π , ...), and imaginary numbers ($\sqrt{-1} = i$). Which of these do you think was the hardest *at the time* for mathematicians to come up with? Justify your thoughts in essay format (1-2 pages), but you may add illustrations if helpful.

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(Extra writing space for essay)



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