MATA02-2022: Quiz 4 Practice [20pts total]

During tutorials week 10: March 21-25
Write your name and Student Number on every single page of work, or else you will not get credit

1. [5pts]

Solve the following division problem in modular arithmetic, or say that there is no unique answer. If there is no unique answer, explain why.

$$12/23 \quad \mathrm{mod} \ 79$$

2. [5pts]

Solve the following power problem in modular arithmetic.

$$7^{9001} \quad \mathrm{mod}\ 59$$

3. [10pts]

Below is an attempted proof. Say whether each line of the proof is correct, assuming all previous lines are correct. Please justify your assertion of whether it is right or wrong for each line.
Note that notationally, I will use "$\equiv$" sometimes without specifying the modulus, but that is not to be considered an error.

Theorem and proof

I claim that if $n^2 \equiv 1 \mod 3$, then $n \equiv 1 \mod 3$.

1. Suppose for the sake of contradiction that
   $n \not\equiv 1 \mod 3$.

2. Then $n \equiv 2 \mod 3$.

3. But by the rules of multiplication in modular arithmetic,
   $n^2 \equiv 2^2 \equiv 4 \equiv 1 \mod 3$.

4. Therefore, to conclude, if $n^2 \equiv 1 \mod 3$, then
   $n \equiv 1 \mod 3$.

Extra space for problem 3