Epidemic modelling basics (1-variable models)
Lecture 12a: 2021-08-06

MAT A35 – Summer 2021 – UTSC

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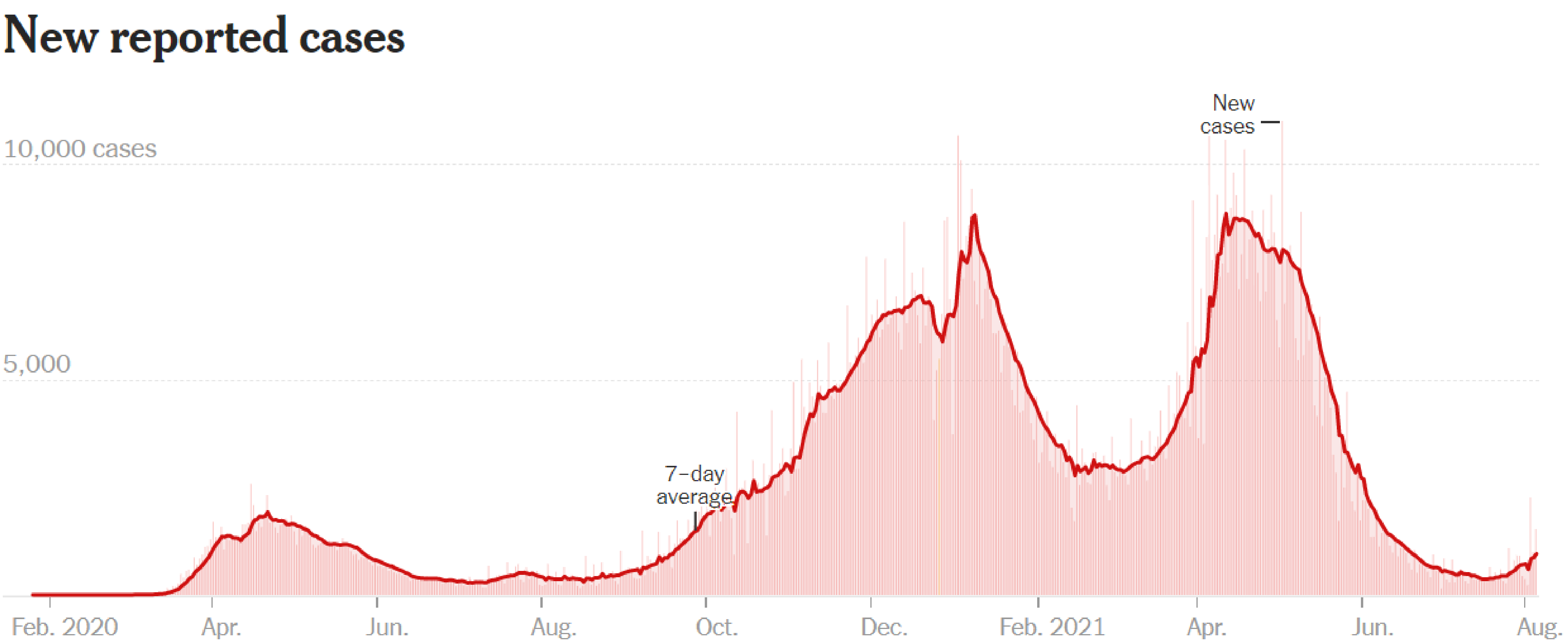
Image credit: https://commons.wikimedia.org/wiki/File:Coronavirus_SARS-CoV-2.jpg

Epidemic curves – Covid19 in Canada



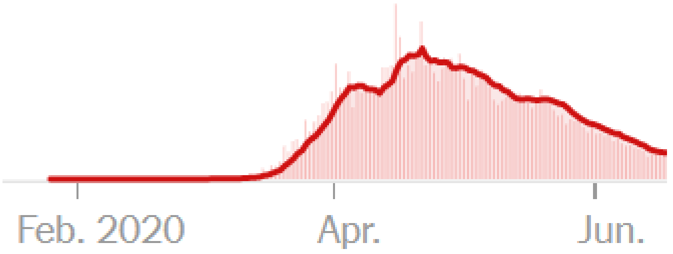
https://www.nytimes.com/interactive/2021/world/canada-covid-cases.html

Infection rates

- Assumption 1: each infected individual infects other individuals at a constant positive rate β.

Exponential model: $I(t) = I(0)e^{\beta t}$

- Let's focus on the initial months of the pandemic.
- Can use regression to find the good values for β, or even just trial and error.



What's wrong with the model?
A: Model is too simple
B: Cannot determine good β
C: Doesn't reflect the data
D: All of the above
E: None of the above

- Model doesn't take into account finite population size.

Compartmental models

- Assumption 2: there is a total fixed population size $N = I(t) + S(t)$, where S is the number of Susceptible individuals.

- Does this fix the problems from the previous slide?

A: Yes
B: No
C: Maybe
D: ???

SI Model of Epidemics

- Modified assumption 1: The infection rate is proportional to the average number of times an infected individual encounters a susceptible individual in the population, assuming random encounters.

Solving SI model qualitatively

- $N = S(t) + I(t)$
- $\dot{S} = -\frac{\beta SI}{N}$
- $\dot{I} = \frac{\beta SI}{N}$

Solving the SI model exactly

- $\dot{I} = \frac{\beta I}{N}(N - I)$
- What methods should we use?

A: Separation of variables
B: Integrating factor
C: u-substitution
D: All of the above
E: None of the above

Integrating factor and u-substitution

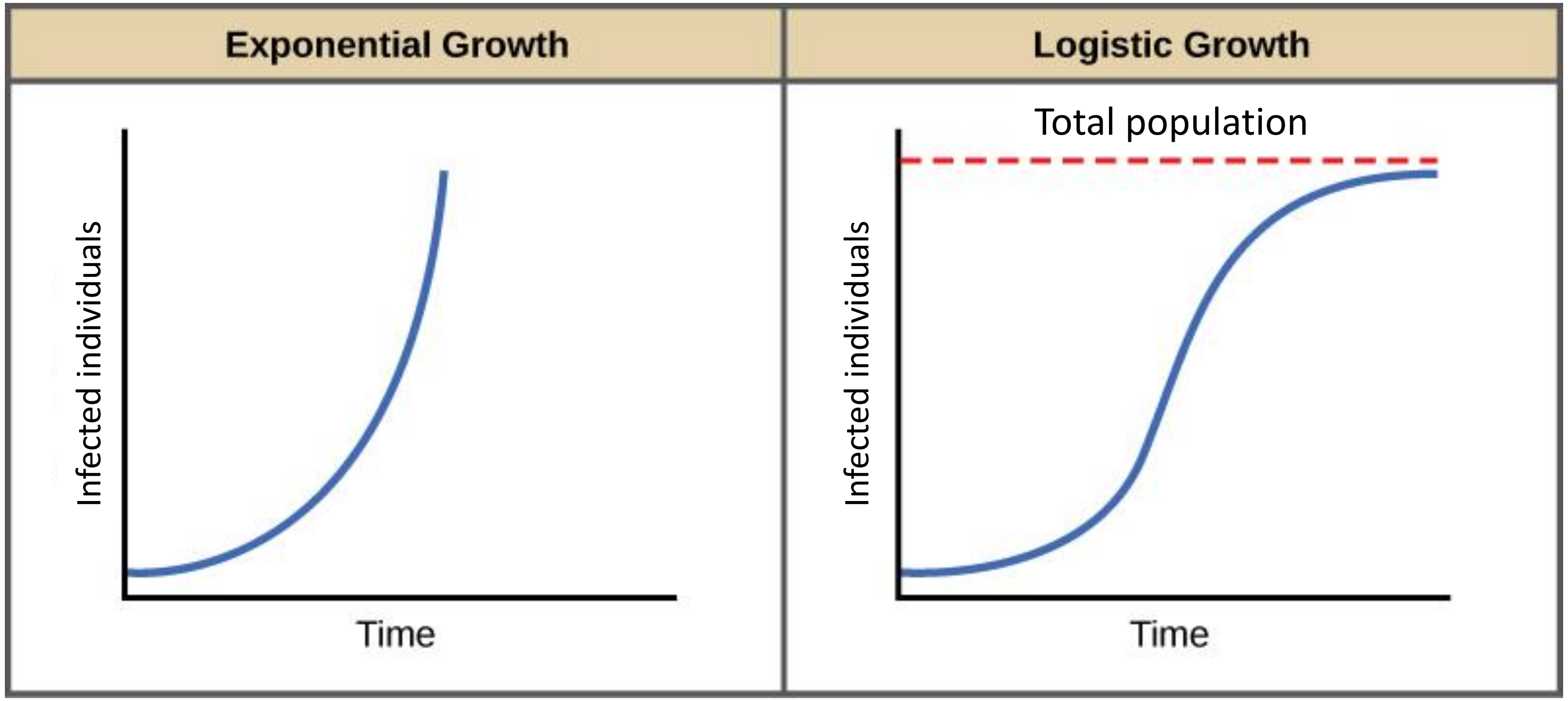
- $\dot{I} = \frac{\beta I}{N}(N - I)$

Separation of variables

- $\dot{I} = \frac{\beta I}{N}(N - I)$

Logistic growth equation

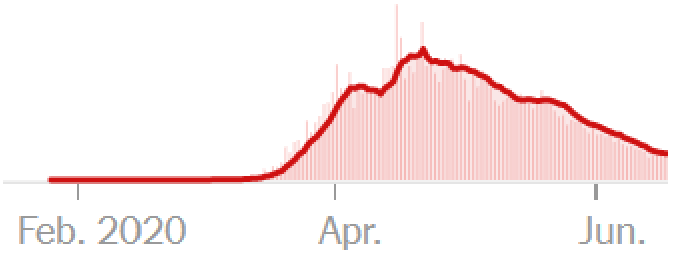
- SI model has logistic growth, which starts out like exponential growth, but levels out as everyone is infected.

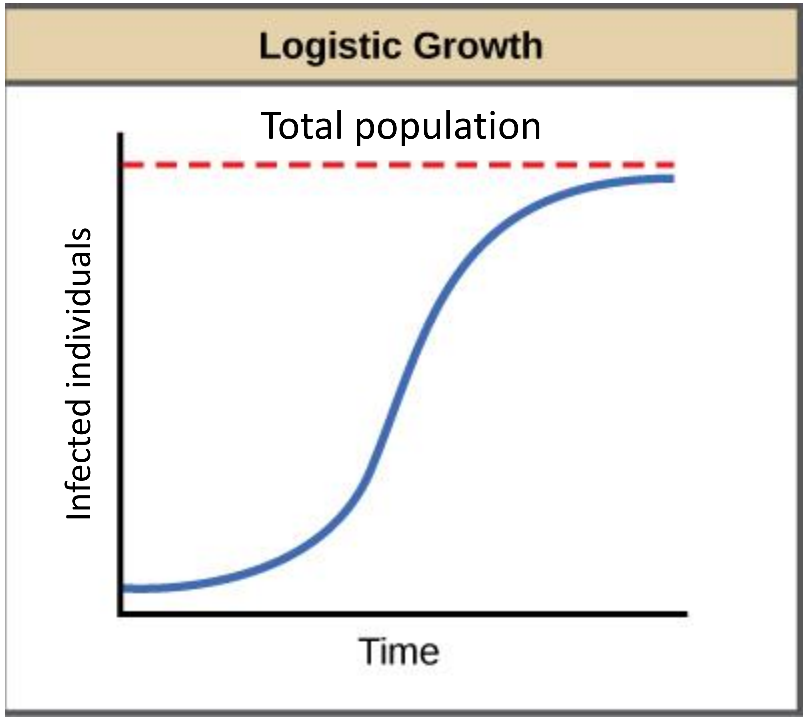


https://commons.wikimedia.org/wiki/File:Figure_45_03_01.jpg

Further improvements

- We now no longer go off to infinity, which is good.
- But, we still are missing the downward part of the epidemic curve.
- How do we get the number of infected to go back down in our model?





A: Add a recovery term
B: Add a mortality rate
C: Add an immunization rate
D: All of the above
E: None of the above

SIS Model

- Assumption 3: Individuals recover at rate γ, and become susceptible to re-infection.

Multiple cases

- $\frac{dI}{dt} = \left(\beta - \frac{\beta I}{N} - \gamma\right) I$
- Note, we have two parameters, β and γ, so there are three cases to consider.
- What are your guesses for behavior in each of the following?
- Case 1: $\beta = \gamma$
- Case 2: $\beta < \gamma$
- Case 3: $\beta > \gamma$

A: Disease dies out
B: Number of infected goes to nonzero constant
C: Number of infected oscillates up and down
D: All of the above
E: None of the above

Case 1: $\beta = \gamma$

- $\frac{dI}{dt} = \left(\beta - \frac{\beta I}{N} - \gamma\right) I$

Case 2: $\beta < \gamma$

- $\frac{dI}{dt} = \left(\beta - \frac{\beta I}{N} - \gamma\right) I$

Case 3: $\beta > \gamma$

- $\frac{dI}{dt} = \left(\beta - \frac{\beta I}{N} - \gamma\right) I$

Basic reproduction number

- Case 1: $\beta = \gamma$. Disease dies out.
- Case 2: $\beta < \gamma$. Disease dies out.
- Case 3: $\beta > \gamma$. Disease persists.
- In the SIS model, we call the ratio $R_0 = \frac{\beta}{\gamma}$ the *basic reproduction number* of the system because it is the number of secondary infections β caused during the infectious period $\frac{1}{\gamma}$.
- When $R_0 > 1$, disease persists.
- When $R_0 \leq 1$, disease dies out.