Volume and improper integration
Lecture 2b – 2021-05-19

MAT A35 – Summer 2021 – UTSC

Prof. Yun William Yu

Volume of simple solids

Invention of pottery?

A: Before 1 AD

B: 1-1000 AD

C: 1000-1500 AD

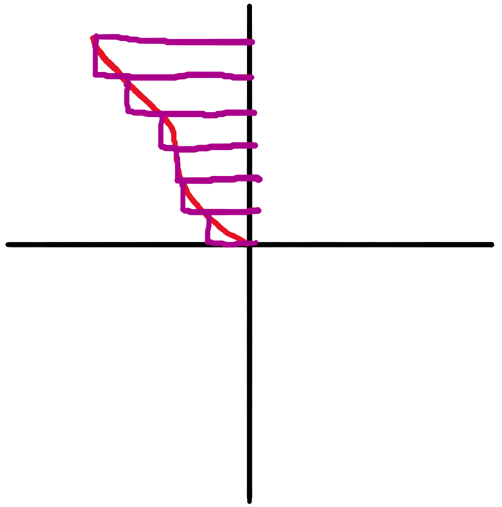
D: 1500-1800 AD

E: 1800 AD-present

Solids of revolution

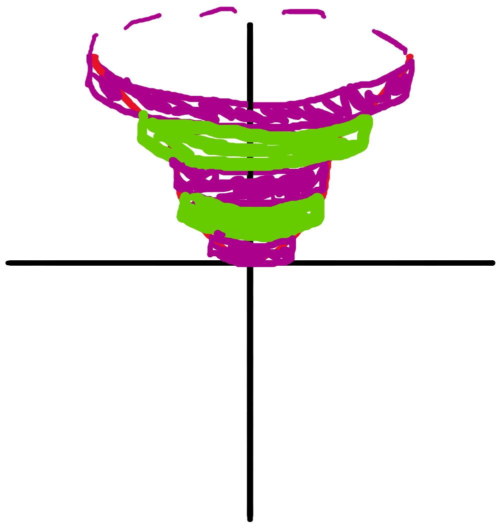
- Area under a curve can be approximated by rectangles

$$A = \lim_{n \to \infty} \sum_{1}^{n} f(x_i)\Delta x$$

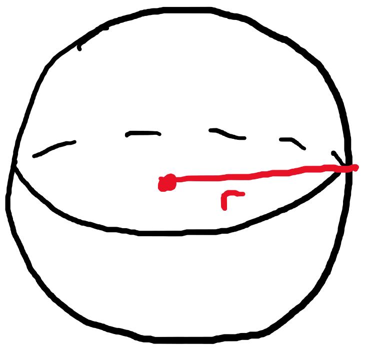


- What if we rotate about the vertical axis? What is the volume?

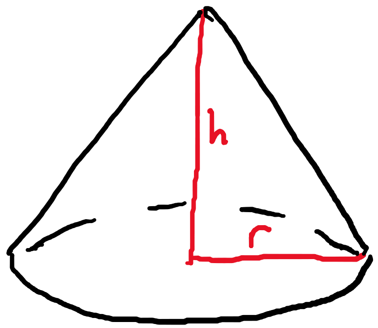
$$V = \lim_{n \to \infty} \sum_{1}^{n} \pi\big(f(x_i)\big)^2 \Delta x$$



Example – Volume of a sphere

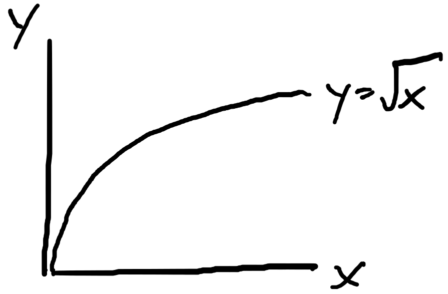


Example – Volume of cone



Try it out

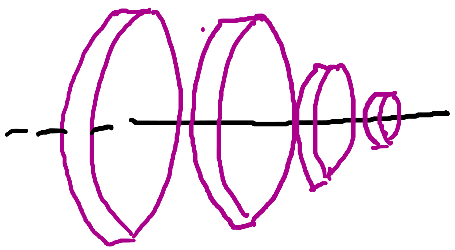
- Find the volume of the solid of revolution generated by rotating the region under the graph of $y = \sqrt{x}$ from $x = 0$ to $x = 1$.



A: $\pi - 1$
B: $\pi/2$
C: $\pi/3$
D: π
E: None

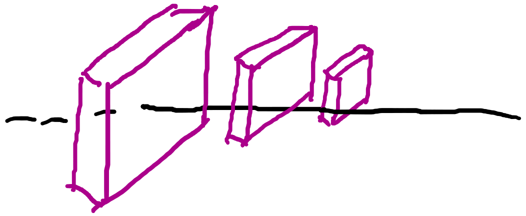
Other Volume Integrals

- Integrating disc volumes along an axis



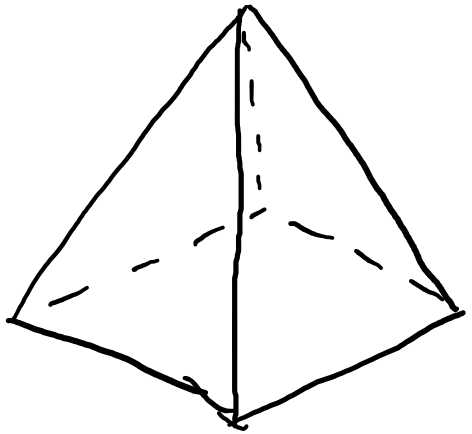
$$\lim_{n\to\infty} \sum_1^n \pi\left(f(x_i)\right)^2 \Delta x$$

- What about other shapes?



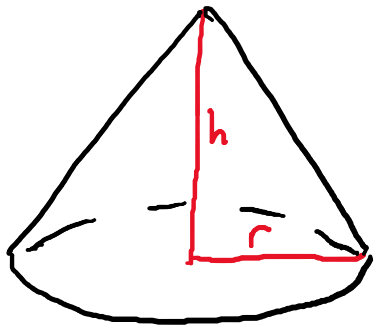
$\lim_{n\to\infty} \sum_1^n A(x)\Delta x$, where $A(x)$ is the area of each slice to be multiplied by Δx.

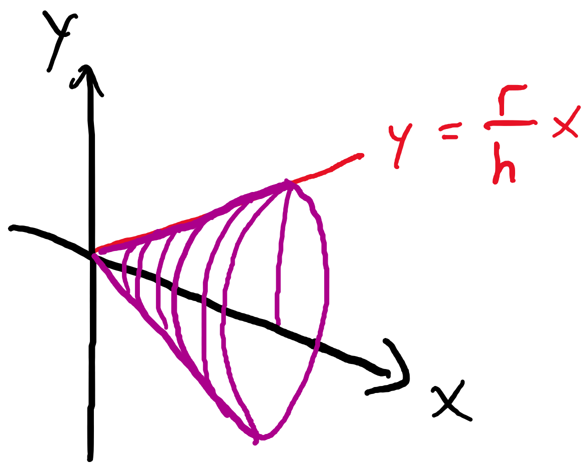
Example - Pyramid



- Suppose the vertical cross section of a pyramid 100 meters tall is always a square, and suppose the side-length of the square is $100 - x$ meters, where x is the height above ground in meters.
- What is the volume of the pyramid?

Surface Areas?





A: True
B: False
C: ???
D: !!!
E: None

- Using discs does NOT work for surface areas because you are incorrectly approximating the paths.