Multilinear and Nonlinear Regression Lecture 6c – 2021-06-16

MAT A35 – Summer 2021 – UTSC

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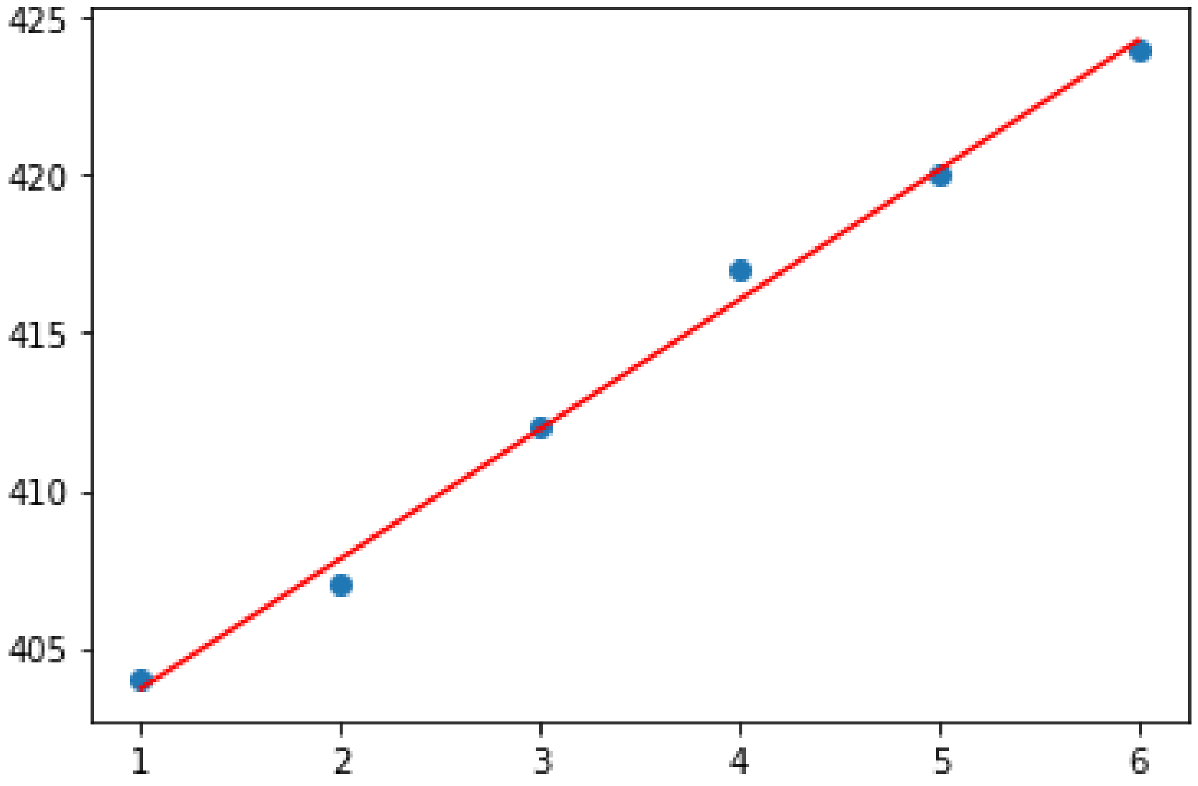
Single variable linear regression

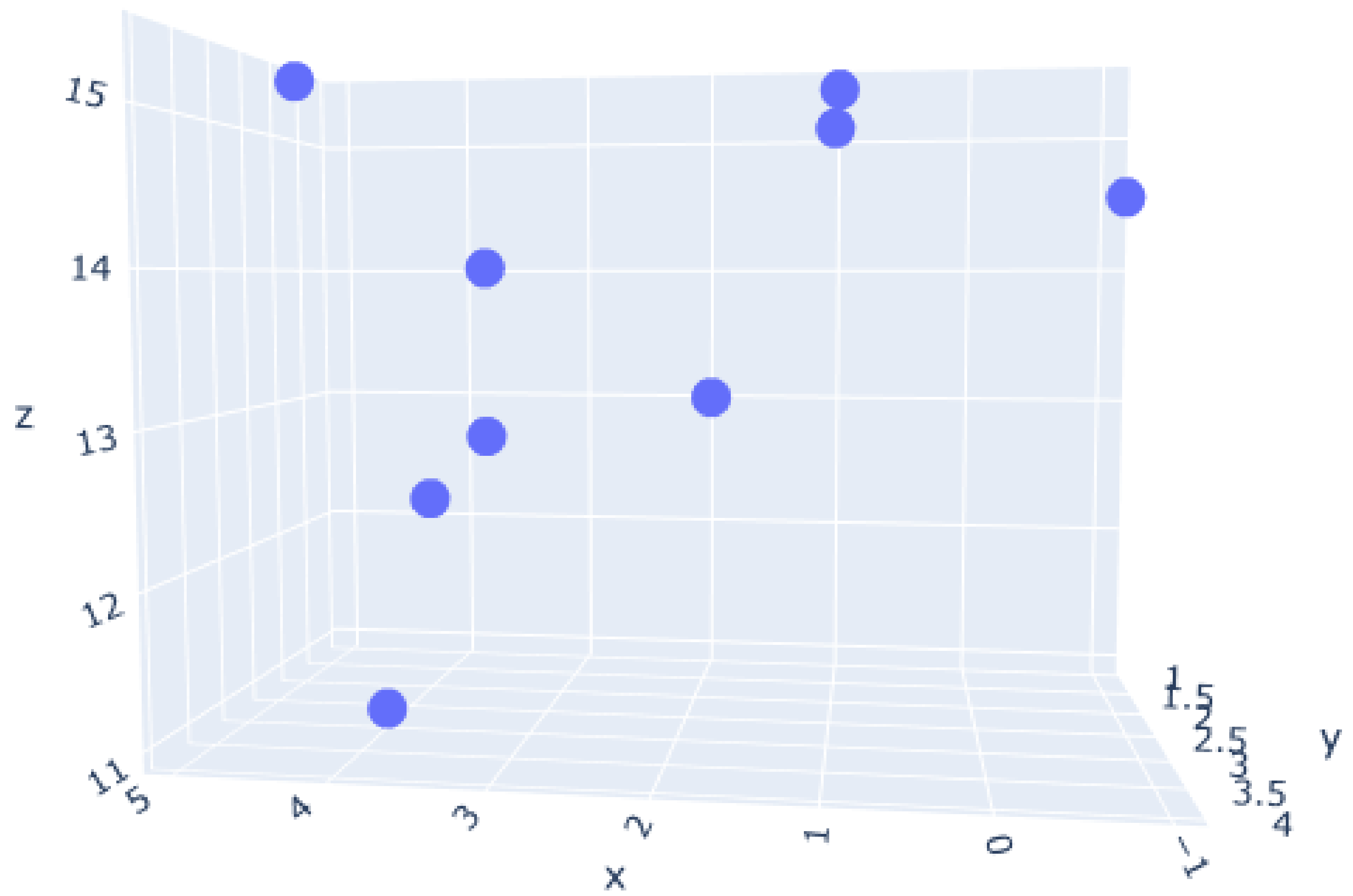
- Given samples of the dependent variable $y_1, \dots, y_n$ at values of the independent variable $x_1, \dots, x_n$, we want to find the linear model $f(x) = mx + b$ such that $y_i \approx f(x_i)$, the "best-fit" line.



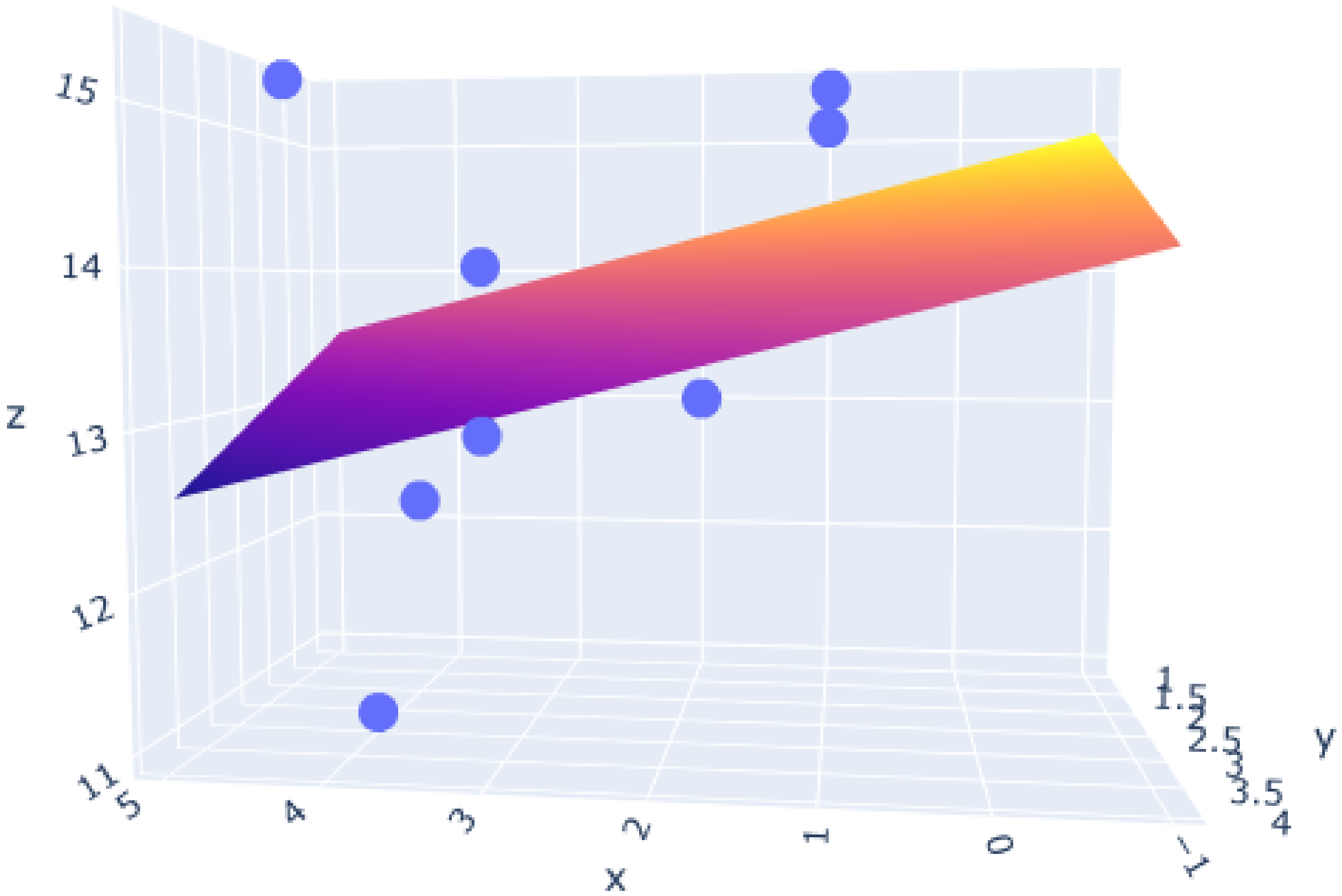
Two-variable linear regression

- What if we have multiple independent variables?
- Suppose we are measuring the water temperature in Lake Ontario, and want to know how the temperature varies as a function of location

3D Scatter Plot of temperatures



Best-fit plane

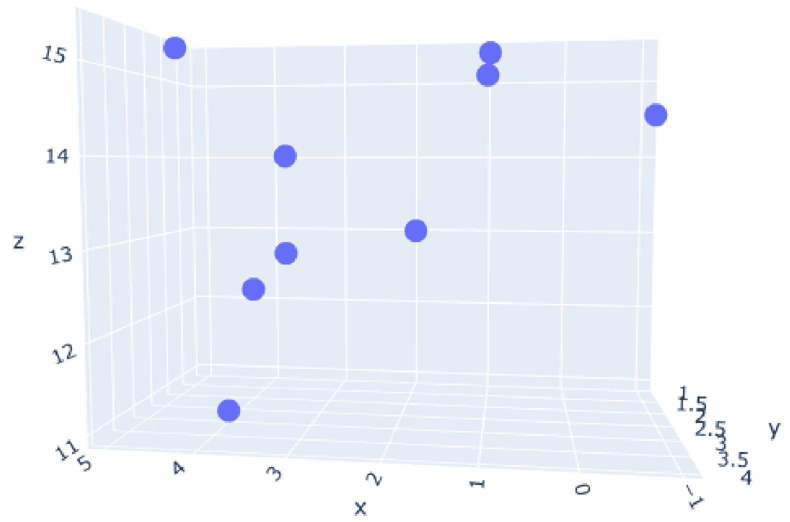


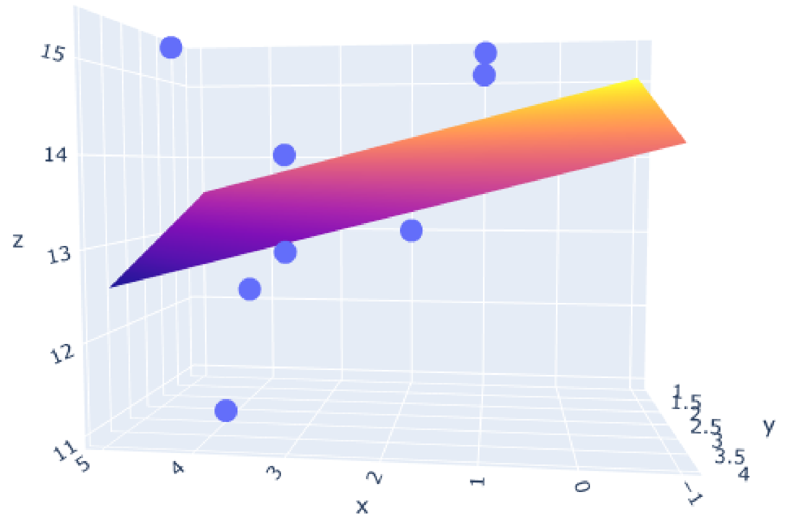
Two-variable linear regression

- Let x and y be the independent variables. Let z be the dependent variable. Given samples $z_1, \dots, z_n$ at values $(x_1, y_1), \dots, (x_n, y_n)$, we want the linear model

$$f(x, y) = m_1 x + m_2 y + b$$

such that $z_i \approx f(x_i, y_i)$, the “best-fit” plane.





Multilinear regression

- One independent variable, one dependent variable

- Two independent variables, one dependent variable

- Many independent variables, one dependent variable

- Can also have many independent variables, many dependent…

Try it out

- You are measuring the temperature of Lake Ontario as a function of location. You get the following data:

Longitude	Latitude	Temperature
76.5 W	43.5 N	12.2
76.5 W	43.9 N	12.1
77.0 W	43.6 N	11.6
77.0 W	43.8 N	11.5
78.0 W	43.3 N	13.7
78.0 W	43.7 N	13.1
79.5 W	43.8 N	12.3
79.5 W	43.9 N	12.1

A: 12.06
B: 12.35
C: 12.54
D: 12.89
E: None of the above

- The GPS coordinates for the lake near Toronto are 43.6 N, 79.3 W. What do you predict the lake water temperature to be near Toronto?

Nonlinear regression

- What if our data doesn't look linear?

Different types of regression

- Linear regression: $f(x) = mx + b$
- Quadratic regression: $f(x) = m_2x^2 + m_1x + b$
- Cubic regression: $f(x) = m_3x^3 + m_2x^2 + m_1x + b$
- Polynomial regression of degree n:

$$f(x) = b + \sum_{i=1}^{n} m_i x^i$$

- Exponential regression: $f(x) = c_1 e^{c_2 x}$
- Power dependencies: $f(x) = c_1 x^{c_2}$

Convert nonlinear to multilinear

Intuition guess

- Linear vs. Quadratic vs Cubic: which model will have smaller Mean Square Error for the following data:

A: Linear
B: Quadratic
C: Cubic
D: Same error for All
E: None of the above

Be careful about too many parameters

- The more parameters you have (e.g. in a polynomial regression), the better your mean squared error will be.
- However, sometimes, you will overfit to the data.
- John von Neumann: “with four parameters, I can fit an elephant, and with five I can make him wiggle his trunk”.

Exponential regression

Power dependencies