Pure-time and separable ODEs Lecture 7b – 2021-06-30

MAT A35 – Summer 2021 – UTSC

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Pure-time ODEs: $y' = f(x)$

- Can solve via integration: $y = \int f(x)dx$

General vs Particular Solutions

- The general solution has some constant(s) in it, and covers all possible particular solutions.
- A particular solution assigns specific values to those constants.

Initial value problem

- When you specify “initial conditions”, you choose a single particular solution out of the general solution.

Try it out

- $\dot{x} = \sin t - 1$
- What is the solution to the initial value problem $x(0) = 0$?

A: $x = \sin t$
B: $x = -\cos t - t$
C: $x = -\cos t - t + 1$
D: $x = -\cos t - t + C$
E: None of the above

Separating the derivative

- Another way to think about pure-time ODEs:
- We can “split” the derivative $\frac{dy}{dx} = f(x)$ by “multiplying” by dx on both sides: $dy = f(x)dx$, and then integrate on both sides.

Separable ODEs

- If we can split a first-order ODE so that one side has all of the dependent variable and the other side has all of the independent variable, then we can integrate on both sides.

Implicit vs explicit solutions

- An explicit solution to an ODE with independent variable x and dependent variable y is of the form $y = f(x)$.

- An implicit solution to an ODE with independent variable x and dependent variable y is of the form $F(x, y) = 0$.
  - Sometimes, an implicit solution is the best we can do.
  - Example on next slide.

Implicit solution initial value problem

Try it out: is the following separable?

- $y' - y^2 x \sin x^2 = 0$
- $\dot{x}x^2 + t^2 e^t = 4$
- $\dot{x}t^2 + x^2 e^x = 4$
- $y' = xy$
- $y' = \sin(xy)$

A: Separable
B: Not separable
C: Cannot tell
D: ???
E: None of the above

Example: $y' = xy$, where $y(1) = 1$

Try it out: $y' = 3x^2e^{2y}$

A: $y = -\frac{1}{2}(-12x + C)$
B: $e^{-2y} = -12x + C$
C: $-\frac{1}{2}e^{-2y} = 6x + C$
D: All of the above
E: None of the above