tags: `MATA35-2021`

Quiz 1 - Practice

Problem 1 (30pts)

Solve each of the following problems. If there are multiple potential answers, give your response in the most general form possible. You do not need to show your work.

1. $\int 5dx$
2. $\int x^3dx$
3. $\int \frac{4}{x+1}dx$
4. $\int e^{-2x}dx$
5. $\int \sin 3xdx$
6. $\int \frac{1}{2}\cos 4xdx$
7. $\int (x^2-2x+1)dx$
8. $\int \frac{1}{(2y+1)(3y+1)}dy$
9. $\int (e^u+\sin u)du$
10. $\int \left[\frac{d}{dx}((\ln(2+\sin x))e^{x^2}+1)\right]dx$

Problem 2 (40pts)

Solve each of the following definite integrals. You should simplify as much as possible without a calculator, but may leave answers in terms of $e, \ln, \sin, \sqrt{\ldots}$, etc. You must show your work.

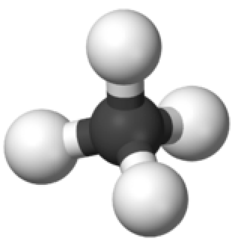
1. $\int_0^1 60x\sqrt{4x+1}dx$
2. $\int_0^1 4xe^{x^2}dx$
3. $\int_{-1}^{\pi-1} \sin(x+1)[\cos(x+1)]^2dx$
4. $\int_1^2 (3\ln x)dx - \int_1^2 (2\ln x)dx$
5. $\int_0^{\pi} (\sin x)e^{-2x}dx + \int_{\pi}^{\infty} (\sin x)e^{-2x}dx$ (very hard; hint: integration by parts twice and combine two sides. Actual quiz problem will be easier.)

Problem 3 (15pts)

Over a 10-day period, the instantaneous rate of rainfall on a lake is measured by $f(t) = te^{-t}$ inches per day, and t is in days since the start of the period. Over that same time period, the lakewater evaporates at a rate of $g(t) = 1 + \sin \pi t$ inches/day—$g(t)$ is nonnegative, but it is a rate of evaporation, so that is water that is removed from the lake. What is change in the water level of the lake after the 10-days? Does it increase or decrease? You must show your work.

Problem 4 (15pts)

You are sitting in an organic chemistry Zoom lecture learning about methane (CH_4) when suddenly you realize that because methane has a tetrahedral shape, you can approximate the volume.

Suppose the height from the base to an opposite vertex is approximately 100 pm (it's actually 145 pm for those of you who are chemists, but 100 is easier to compute with).

You decide that instead of solving for the volume analytically, you want to use a numerical approximation. As such, you measure the area of the triangle cross-section as a function of height at 3 points:

- $A(0) = 6000$
- $A(50) = 1500$
- $A(100) = 0$

1. Approximate the area using Riemann sums.
2. Approximate the area using the trapezoid rule.
3. Which of the two approximations do you think is more accurate, and why?

You must show your work.