tags: `MATA35-2021`

Quiz 1 - Wednesday - Solutions

Problem 1 (40pts)

Solve each of the following problems. If there are multiple potential answers, give your response in the most general form possible.

1. $\int 4dx = 4x + C$
2. $\int 2x^{-2}dx = -\frac{2}{x} + C$
3. $\int \frac{9}{2x+1}dx = \frac{9}{2}\ln|2x+1| + C$
4. $\int \frac{1}{4}\sin(x-1)dx = -\frac{1}{4}\cos(x-1) + C$
5. $\int (2x^2 - 2x - 1)dx = \frac{2}{3}x^3 - x^2 - x + C$
6. $\int \frac{1}{(y-1)(2y-1)}dy = \ln|y-1| - \ln|2y-1| + C$
7. $\int (e^{2u} + \cos u)du = \frac{1}{2}e^{2u} + \sin u + C$
8. $\int \left[\frac{d}{dx}((\ln x^2)(2+\sin x)e^{x^2} + 3)\right]dx = (\ln x^2)(2+\sin x)e^{x^2} + C)$

Problem 2 (30pts)

Solve each of the following definite integrals. You should simplify as much as possible without a calculator, but may leave answers in terms of $e, \ln, \sin, \sqrt{\dots}$, etc.

> Each problem is worth 15 points.

1. $\int_0^\pi \sin(x)[\cos(x) - 1]^2 dx$

> Take off additional points for arithmetic errors, but don't penalize twice. i.e. if they made a silly error early on, but got the rest of the problem right, give them most of the later credit.

> Option 1 (change the limits)
>
> - u-substitution $u = \cos x - 1, du = -\sin x dx$
> - changing the limits to $u = 0, -2$: $-\int_0^{-2} u^2 du$
> - solving to get $\frac{8}{3}$

> Option 2 (solve the antiderivative first and change back to x's)
>
> - u-substitution $u = \cos x - 1, du = -\sin x dx$
> - solving to get $-\frac{1}{3}u^3 = -\frac{1}{3}(\cos x - 1)^3$
> - plugging in the original limits to get $\frac{8}{3}$.

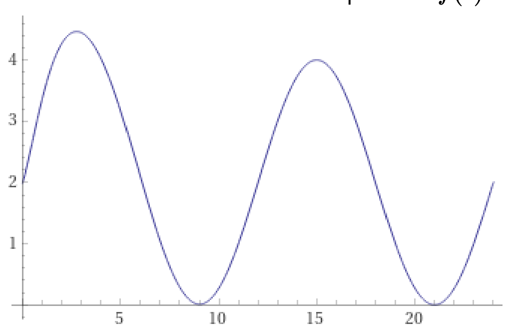
2. $\int_0^\pi xe^{-2x}dx + \int_\pi^\infty xe^{-2x}dx$

> - recognizing that the integrals can be combined to just $[0, \infty]$
> - integration by parts using $u = x, du = dx$ and $dv = e^{-2x}, v = -\frac{1}{2}e^{-2x}$. Should get $-\frac{x}{2}e^{-2x} - \frac{1}{4}e^{-2x}$
> - solving the improper integral by "plugging-in" 0 and ∞ to get $\frac{1}{4}$

Problem 3 (15pts)

Every day, the Gibson coal-fired power plant in Owensville, Indiana, USA produces about 50 kilotons of CO_2 into the atmosphere.

Suppose you set up a post-combustion carbon capture system set up. Starting from midnight, the instantaneous rate of carbon capture is $f(t) = t^2 e^{-t} + 2 + 2\sin(\frac{t\pi}{6})$ kilotons/hour of CO_2.



After carbon capture, how much CO_2 would be released by the power plant into the air each day? You may approximate $e^{-24} \approx 0$.

- recognizing that total carbon capture per day is $\int_0^{24} f(t)dt$.
- $\int_0^{24} 2dt = 48.$
- $\int_0^{24} 2\sin(t\pi/6)dt = 0$
- $\int_0^{24} t^2 e^{-t} dt = 2 - \frac{626}{e^{24}} \approx 2$
- 50 - (48 + 2) = 0. So just about all of the CO_2 is captured.

Problem 4 (15pts)

You are King Arthur's court wizard. Your job is to determine the average airspeed of an unladen African swallow, but you only have one swallow. You decide to send it on a 4-hour round-trip journey, and you measure its speed once an hour, for a total of 5 measurements (including the 0th-hour measurement).

- $v(0) = 15$ miles per hour
- $v(1) = 20$ miles per hour
- $v(2) = 18$ miles per hour
- $v(3) = 23$ miles per hour
- $v(4) = 19$ miles per hour

> Part 1 is worth 6 points, part 2 is worth 9 points. Note that getting average speed by just taking the 5 speed measurements and averaging them is INCORRECT for the purposes of this problem.

1. Approximate the distance traveled using rectangular Riemann sums. Use this to compute the average speed.

> May either use right- or left-rectangular Riemann sums.

> **Right Riemann sums**:
>
> - total distance = (1 hour)(20+18+23+19 mph) = 80 miles.
> - average speed = 80 miles / 4 hours = 20 mph

> **Left Riemann sums**:
>
> - total distance = (1 hour)(15+20+18+23 mph) = 76 miles.
> - average speed = 76 miles / 4 hours = 19 mph

2. Approximate the distance traveled using the trapezoid rule. Use this to compute the average speed.

Trapezoid rule requires adding all datapoints times two, except for the first and last ones. Then divide by two.

- (1 hour)[15 + 2(20+18+23)+ 19 mph]/2 = 156/2 = 78 miles (note that remembering the multiplication/division by 2 for the trapezoid rule is worth 3 pts)
- average speed = 78 miles / 4 hours = 19.5 mph.