tags: MATA35-2021

Quiz 2 - Wednesday - Solutions

Problem 1: Matrix operations [25pts]

Solve each of the following problems. If the answer is undefined, state so explicitly.

1. $$\begin{bmatrix} -1 & -2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

> $$\begin{bmatrix} -5 \\ 11 \end{bmatrix}$$

2. $$\begin{bmatrix} -1 & 2 & 0 \\ 5 & 0 & 2 \end{bmatrix} \begin{bmatrix} 9 & 1 & 2 \\ 3 & -3 & 0 \end{bmatrix}$$

> Undefined, because dimensions don't line up for matrix multiplication.

3. $$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 \end{bmatrix}$$

> $$\begin{bmatrix} 4 & 5 \\ 8 & 10 \\ 12 & 15 \end{bmatrix}$$

4. $$\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

> Undefined because dimensions don't line up for addition/subtraction.

5. $$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^5 \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

> $$\begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

Problem 2: Matrix inverse and determinants [25pts]

Find the determinant and multiplicative inverse for each of the following matrices. If an inverse does not exist or is undefined, say why.

1. $$\begin{bmatrix} -1 & 2 \\ 3 & -6 \end{bmatrix}$$

> $\text{determinant} = 0$, so the inverse does not exist.

1. $$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$\text{determinant} = 1$

$$\text{inverse} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

Problem 3: Eigendecomposition [25pts]

Find all eigenvalues and eigenvectors for the following matrix. Show your work.

1. $$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

$\lambda_1 = 0, v_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$\lambda_2 = 5, v_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Problem 4: Leslie matrix word problem [25pts]

As a backyard ecologist, you have been keeping track (as best you can) of the dandelion population in your yard. Each year, dandelions pop up from two sources:

1. New seedlings (S): these are new grown plants that grow from the tens of thousands of seeds that are released.
2. Old taproots (T): these are old plants whose root systems have survived the winter and have come back as perennials.

Because of the short time to seed, even new seedlings will produce seeds over the course of a year, so both new seedlings and old taproots will produce seeds.

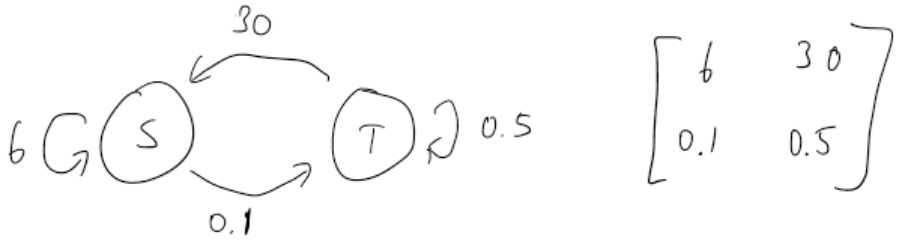
After careful counting, you decide to build a model off the following observations.

- Each year, a seedling has a 0.1 probability of surviving to the next year.

- Each year, a seedling produces 6 new seedlings for the following year.
- Each year, old taproots have a 0.5 probability of surviving to the next year.
- Each year, old taproots produce 30 new seedlings for the following year.

Answer the following questions:

1. Write a Leslie matrix and Leslie diagram for this age-structured model.



2. Suppose your yard starts with 65 seedlings and no taproots this year. How many seedlings and taproots will you have after 10 years time? (you do not need to evaluate powers/exponents and can leave your answer unreduced)

$(6.5)^{10}\begin{bmatrix}60\\1\end{bmatrix}$, so $6.5^{10}\cdot 60$ seedlings and 6.5^{10} taproots.

3. What is the long-term ratio of seedlings to taproots?

60 seedlings for each taproot.