tags: `MATA35-2021`

Quiz 3 - Practice Problems

Problem 1: Multivariable functions

Consider the function

$$f(x, y) = (x^4 - 4x^2)e^{-0.1y^2}$$

Graph this function at several different zoom levels and describe the behavior of the function in words.

Do you see any local extrema?

Problem 2: Partial derivatives

Compute all of the 1st and 2nd-order partial derivatives of the function $f(x, y)$ from above.

Problem 3: Maximums and minimums

1. Find all critical points of $f(x, y)$ using the partial derivatives from Problem 2.
2. Use the various derivative tests we learned to classify each critical point as either a minimum or maximum if possible. If it is not possible, explain why. Note that one critical point will be unclassifiable using the deriviative tests.
3. In lecture, we defined a local maximum of $f : \mathbb{R}^2 \to \mathbb{R}$ as a point (x_0, y_0) where there exist a small neighborhood N around (x_0, y_0) where $f(x, y) < f(x_0, y_0)$ for any $(x, y) \in N$. This definition is usually actually referred to as the definition of a **strict** local maximum.
   Often, we relax the definition to replace the "$<$" with a "$\leq$"; i.e. a "non-strict" local maximum is a point where $f(x, y) \leq f(x_0, y_0)$ for any $(x, y) \in N$. Use these two descriptions to characterize the behavior of the unclassifiable critical point from 3.2 above. Is it a strict local maximum? Is it a non-strict local maximum? Justify your claims.

Aside: The usual definition (such as found in your textbook) of local minimums and maximums actually is the non-strict one. The use of "$<$" and "$>$" in the notes instead of "$\leq$" and "$\geq$" is nonstandard.

Problem 4: Multivariable integration

Find the volume of a solid capped by the surface $g(x, y) = \sqrt{xy} + 1$ bounded by the following inequalities:

- $x \leq 2$
- $y \geq 0$
- $y \leq x^2$
- $y \geq x$

Give your answer to 3 decimal places. Show all of your work, but you may use a calculator for the final computation.

Problem 5: Linear Regression Analysis

The table below shows the average salary of an NFL player player in millions over a 10-year period:

Year	Salary in millions
1992	0.48
1993	0.67
1994	0.63
1995	0.72
1996	0.79
1997	0.74
1998	0.99
1999	1.06
2000	1.12
2001	1.10

1. Use linear regression to find the best fit line.
2. What does your model predict to be the average salary in 2000? Is the prediction good?
3. Predict the average salary in 2017. Is the prediction good?
4. The actual average salary in 2017 was 2.7 million. If your prediction was significantly off, explain in words why that might have happened.

Problem 6: Nonlinear regression

Attempt to build a better model by using nonlinear regression.

1. What are the predicted average salaries in 2000 vs. 2017 using quadratic regression? Do these predictions make sense?
2. What are the predicted average salaries in 2000 vs. 2017 using cubic regression? Do these predictions make sense?
3. Importantly, note that although the cubic model is better fit to the data in the table (e.g. 2000), it seems to give a worse prediction for 2017. What happened?