tags: `MATA35-2021`

Quiz 3 - Take-home

This is a 24-hour take-home quiz. It will be released on Crowdmark Friday, July 9 at 2pm, and will be due Saturday, July 10 at 2pm.

- You may not collaborate with each other on Quiz 3.
- You may not ask questions on external online forums (such as StackExchange, Kaggle, Chegg, etc.).
- You may ask clarification questions on the MATA35 class Piazza for me or your TAs to answer. This is to ensure that everyone has access to the same information.
- You may use the textbook, as well as general online math resources, so long as they are not specific to the problems on Quiz 3.
  e.g. You may look up a tutorial on how to do linear regression using Python, but if you come across one of the quiz problems on Chegg, you may not use the provided specific solution (indeed, I'd appreciate if you let me know if you encounter the exact quiz question on Chegg).
- You may make full use of calculators (online or physical). However, some problems may ask you to show your work, in which case you cannot simply answer that the calculator solved it for you. Some online calculators and computer algebra systems are extremely powerful and can fully solve some of the quiz problems. However, you should show still your work as if you solved it with only the help of an ordinary scientific calculator.
- Problems 1 through 4 should be submitted to Crowdmark as usual.
- For problems 5 and 6, you should write a report including all of your code, as well as your free-form answers to the questions. I recommend writing a report using the Markdown features of Google Colab (https://colab.research.google.com (https://colab.research.google.com)) / Jupyter Notebook via UToronto Syzygy (https://utoronto.syzygy.ca (https://utoronto.syzygy.ca)), such as found in the Practice Quiz 3 solutions guide:
  (https://colab.research.google.com/drive/18tze1JA0_u4v6mL1L5oOouFvgKiK9JVC?usp=sharing (https://colab.research.google.com/drive/18tze1JA0_u4v6mL1L5oOouFvgKiK9JVC?usp=sharing))

Problem 1: Multivariable functions [10pts]

Consider the function

$$f(x, y) = (x - y)(y - 1)(x + 1)$$

Graph this function at several different zoom levels and describe the behavior of the function in words.

1. **[5pts]** Do you see any local extrema or saddle points? Where are they?
2. **[5pts]** What is the behavior of the function as you go away from the origin in different directions? You should describe the behavior as you go in each direction along the axes, as well as along each of the diagonal directions.

Please include images of the function in your report, illustrating your description above.

Problem 2: Partial derivatives [12pts]

Compute all of the 1st and 2nd-order partial derivatives of the function $f(x, y)$ from Problem 1 by hand.

Note: You must show all of your work for computing the derivatives by hand. You may however use a calculator / online resources to double check your work, which I encourage because Problem 3 depends on the solutions to Problem 2.

Note: Make sure you explicitly mark each of your steps. You will not get credit if you do not show your work.

Problem 3: Maximums and minimums [24pts]

1. **[8pts]** Find all critical points of $f(x, y)$ from Problem 1 using the partial derivatives from Problem 2.
   Sanity Check: did you get all the critical points you saw in Problem 1?
2. **[16pts]** Use the various derivative tests (e.g. eigenvalues of the Hessian or the D-test from the book) we learned to classify each critical point as either a minimum, maximum, or saddle point. If it is not possible to classify, explain why.

Problem 4: Multivariable integration [14pts]

Integrate $g(x, y) = \frac{-x^3+2x^2+x}{(y+1)^2}$ over the region bounded by the following inequalities:

- $y \leq -x^2 + 2x$
- $y \geq 0$
- $y \geq x - 1$

Give your answer to 3 decimal places. Show all of your work, but you may use a calculator for evaluating complicated algebraic expressions. Make sure to explain **in words** each of the steps you are doing.

Note: You must write out all the work for doing the double integration by hand. You will not get credit if you simply plug in the double integral into an online calculator.

Note: the function $g(x, y)$ may be negative in some places, so the interpretation as volume doesn't make as much sense.

Problem 5: Linear Regression Analysis [20pts]

For problems 5 and 6, please include your Python code and any generated graphs. Note that some of the answers are qualitiative analyses, with more than one right answer, and they will be marked on the overall quality of your reasoning.

On Crowdmark, note that we have combined questions 5 and 6 into a single response to make it easier for you to submit. I also encourage you to additionally submit a link to your code within your PDF report if you used a Google colab, but this is not required.

Baby names vary in frequency over time. The table below shows the relative frequencies of the names "Holly" and "Henry" as a function of time from the US Social Security records. The numbers are in units of number per 1000 baby girls for "Holly" and number per 1000 baby boys for "Henry".

Year	Holly	Henry
1940	0.063	5.749
1945	0.284	4.811
1950	0.475	3.835
1955	1.248	2.966
1960	1.240	2.293
1965	1.836	2.006
1970	2.693	1.638
1975	3.235	1.282
1980	2.968	1.113
1985	3.037	1.071

1. **[5pts]** Use linear regression to find the best fit line for each name.
2. **[5pts]** What does your model predict to be the frequencies of each name in the year 1985?
3. **[5pts]** Predict the frequencies of each name in the year 2005.
4. **[5pts]** Without making reference to the actual values given in problem 6, explain why your predictions are likely to be good or bad.

Problem 6: Nonlinear regression [20pts]

Consider the updated data below.

Year	Holly	Henry
1990	1.837	1.039
1995	1.325	1.253
2000	0.848	1.492

1. **[5pts]** Build a model that incorporates the updated data. Note that a linear model may no longer be appropriate, so you may wish to consider using nonlinear regression.
2. **[5pts]** Justify your choice of model(s) above.
3. **[5pts]** Predict the frequencies of each name in the year 2005 using your model. Note that the actual values are 0.510 and 1.964 for Holly and Henry respectively. How good was your model?
4. **[5pts]** Describe some of the remaining shortcomings of your model.