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Quiz 5 - Practice Problems

Problem 1: Higher-order inhomogeneous equation

Consider the following ODE:

$$y'''' + 2y''' + 2y'' + 2y' + y = 1 - e^x$$

1. Find the real homogeneous solution.
2. Find a real particular solution.
3. Find the real general solution.

Problem 2: System of equations

Solve the following initial value problem:

$$\dot{x} = 2x - y + t$$
$$\dot{y} = -x + 2y$$

where $x(0) = \frac{13}{9}$ and $y(0) = -\frac{4}{9}$

Problem 3: Matrix equations

1. Given unknown functions $x(t)$ and $y(t)$, find the general solution to the system:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2. Let $\dot{z} = Az$ for each of the following 2x2 matrices A. Classify the equilibrium at the origin by type and stability.

   ○ $\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$

   ○ $\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

- $\begin{bmatrix} -3 & 1 \\ 0 & -1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$

Problem 4: Word problem

Chemicals enter a house's basement air at a rate of 0.1 mg per minute. Let $F(t)$ and $B(t)$ denote the total amount of chemical present in the first-story air and the basement air after t minutes respectively.

Both the first floor and the basement have volumes of 200 (m^3) each. Air flows from the basement into the first floor at a rate of 2 (m^3) per minute, while air flows from the first floor to the outside at the rate of 4 (m^3) per minute. Air from the outside (with no chemicals present) replenishes the air in both rooms to keep the volumes constant.

1. Draw a 2-compartment model for (B) and (F).
2. Write a system of two first-order differential equations modelling the system.
3. Find the equilibrium values for (B) and (F). (Recall: if a system is at its equilibrium values, then there is no change over time in any of its variables.)