

Quiz 5 - Thursday

Problem 1: Higher-order inhomogeneous equation [25pts]

Consider the following ODE:

$$y''' + y'' + y' + y = 4 + 8e^x$$

1. **[10pts]** Find the real homogeneous solution.
2. **[10pts]** Find a real particular solution.
3. **[5pts]** Find the real general solution.

Problem 2: System of equations [25pts]

Solve the following initial value problem:

$$\begin{aligned}\dot{x} &= x + 3y \\ \dot{y} &= x + 3y\end{aligned}$$

where $x(0) = 5$ and $y(0) = 1$.

Hint: You learned two different methods for solving this problem, and you may use whichever one is easier.

Problem 3: Matrix equations [24pts + 3pts bonus]

Let $\dot{z} = Az$ for each of the following 2x2 matrices A . Classify the equilibrium at the origin by type and stability. **Bonus:** if the type is a node, further specify if it is proper, improper, or neither.

1. $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

2. $\begin{bmatrix} -1 & -1 \\ 9 & -1 \end{bmatrix}$

3. $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$$4. \begin{bmatrix} 0 & 4 \\ -1 & 0 \end{bmatrix}$$

Problem 4: Word problem [26pts]

Tank A contains 100 gallons of pure water. Tank B contains 200 gallons of water with 50 lb of salt dissolved. Water is pumped from tank A to tank B at a rate of 5 gallons per minute. Water is also drained from tank A to the outside at a rate of 5 gallons per minute. Water is pumped from tank B to tank A at a rate of 10 gallons per minute. Pure water is added to tank B to keep the total volume constant.

Let $A(t)$ and $B(t)$ denote the total amount of salt present in tanks A and B respectively.

1. **[10pts]** Draw a 2-compartment model for A and B .
2. **[10pts]** Write a system of two first-order differential equations modelling the system.
3. **[6pts]** Find the equilibrium values for A and B .