

## Quiz 5 - Wednesday

---

### Problem 1: Higher-order inhomogeneous equation [25pts]

---

Consider the following ODE:

$$y''' + y'' - y' - y = x + 1$$

1. **[10pts]** Find the real homogeneous solution.
2. **[10pts]** Find a real particular solution.
3. **[5pts]** Find the real general solution.

### Problem 2: System of equations [25pts]

---

Solve the following initial value problem:

$$\begin{aligned}\dot{x} &= x + 2y \\ \dot{y} &= 2x + 4y\end{aligned}$$

where  $x(0) = 5$  and  $y(0) = 0$ .

**Hint:** You learned two different methods for solving this problem, and you may use whichever one is easier.

### Problem 3: Matrix equations [24pts + 3pts bonus]

---

Let  $\dot{z} = Az$  for each of the following 2x2 matrices  $A$ . Classify the equilibrium at the origin by type and stability. **Bonus:** if the type is a node, further specify if it is proper, improper, or neither.

1.  $\begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$
2.  $\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$
3.  $\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$

$$4. \begin{bmatrix} 0 & 4 \\ -1 & 0 \end{bmatrix}$$

## Problem 4: Word problem [26pts]

---

Tank A contains 100 gallons of pure water. Tank B contains 200 gallons of water with 50 lb of salt dissolved. Water is pumped from tank A to tank B at a rate of 10 gallons per minute. Water is pumped from tank B to tank A at a rate of 5 gallons per minute. Water is additionally drained from tank B to the outside at a rate of 5 gallons per minute. Pure water is added to tank A to keep the total volume constant.

Let  $A(t)$  and  $B(t)$  denote the total amount of salt present in tanks A and B respectively.

1. **[10pts]** Draw a 2-compartment model for  $A$  and  $B$ .
2. **[10pts]** Write a system of two first-order differential equations modelling the system.
3. **[6pts]** Find the equilibrium values for  $A$  and  $B$ .