Multivariable integration Lecture 5c: 2023-02-06

MAT A35 – Winter 2023 – UTSC

Prof. Yun William Yu

Double definite integrals

$$\int_a^b f(x)dx$$

$$\int_a^b \int_c^d f(x,y)\,dx\,dy$$

Definite integration removes the variable

Multiple definite integrals example

$$\int_{y=a}^{y=b} \int_{x=c}^{x=d} f(x, y)\, dx\, dy$$

- First integrate the inside integral, assuming all other variables are constant.
  - $\int_{x=c}^{x=d} f(x, y)\, dx = g(y)$ (because we got rid of the "x")
- Then integrate the outside variable, to get an answer
  - $\int_{y=a}^{y=b} g(y)\, dy =$ answer

Ex: $\int_{0}^{2} \int_{-1}^{2} 10xy^2 dx \; dy$

Try it out

- $\int_0^2 \int_{-1}^1 2y dx\, dy$

A: 2
B: 4
C: 8
D: 16
E: None

Switching order of integrals?

- Last slide: $\int_0^2 \int_{-1}^1 2y dx\, dy$
- What about: $\int_{-1}^1 \int_0^2 2y dy\, dx$

- Often, we can switch the order of integration and get the same answer, but this is not always true.

Variables in the limits of integration

- We can use the outside integral variable in the limits of the inside variable (but not the other way around).

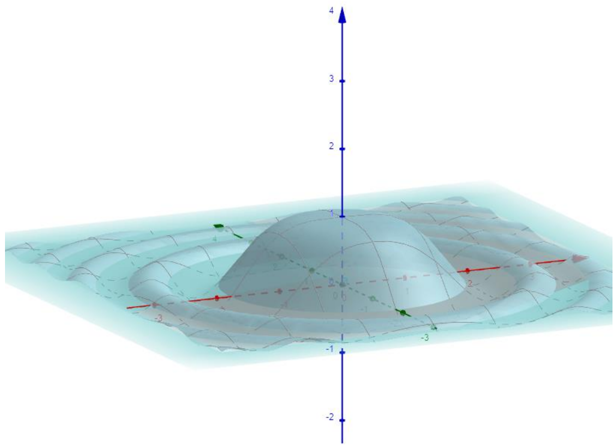
Try it out

- $\int_0^2 \int_1^{y^2} 2x dx\, dy$

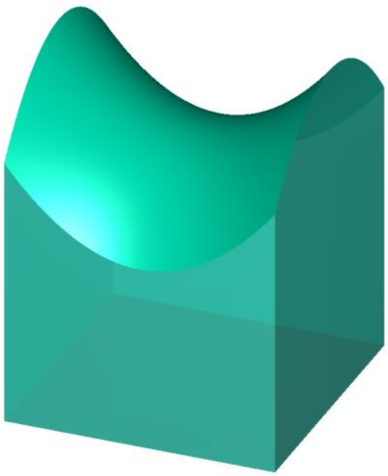
A: $\frac{11}{5}$
B: $\frac{22}{5}$
C: $\frac{33}{5}$
D: $\frac{44}{5}$
E: None

Geometric interpretation

- Multivariable functions $f: \mathbb{R}^2 \to \mathbb{R}$ can be thought of as surfaces.



- Double integrals correspond to the *volume* under the surface for a particular region



https://en.wikipedia.org/wiki/Multiple_integral