Pure-time and separable ODEs Lecture 7b: 2023-02-27

MAT A35 – Winter 2023 – UTSC Prof. Yun William Yu

Pure-time ODEs: y' = f(x)

• Can solve via integration: $y = \int f(x) dx$

y'= 2x15 Exe. $Y = \sqrt{2 \times 15} dx$ y = x 2 + 5x + C general sol $\begin{aligned} \dot{f}_{x} &= sht - 1 \\ \frac{dx}{dt} &= sht - 1 \\ \frac{dt}{dt} &= sht - 1 \\ x &= \int dx &= \int (shrt - 1) dt \end{aligned}$ x= - cost - t + C 6

General vs Particular Solutions

- The general solution has some constant(s) in it, and covers all possible particular solutions.
- A particular solution assigns specific values to those constants.

$$y' = 2 \times 48$$

$$y' = 2 \times 48$$

$$y' = x^{2} + 5 \times 40$$

$$y = x^{2} + 5 \times 41$$

$$y = x^{2} + 5 \times 42$$

$$y = x^{2} + 5 \times 42$$

Initial value problem

• When you specify "initial conditions", you choose a single particular solution out of the general solution.

J phy M Fx. $y = x^2 + 5x + C$ Initial value (IV): y(1) = 10Toplus in y = 10 = 1 + 5 + C =) C=4 =) $y = x^2 + 5x + 4$ particles sol

Try it out

• $\dot{x} = \sin t - 1$

• What is the solution to the initial value problem x(0) = 0? t = -1? $x = -\cos t - t + C$ 0 = -1 + C = -1 + C $x = -\cos t - t + 1$

$$0 = -\cos 0 - 0 + 1$$

 $0 = -1 - 0 + 1$

A: $x = \sin t$ B: $x = -\cos t - t$ C: $x = -\cos t - t + 1$ D: $x = -\cos t - t + C$ E: None of the above

Separating the derivative

- Another way to think about pure-time ODEs:
- We can "split" the derivative $\frac{dy}{dx} = f(x)$ by "multiplying" by dx on both sides: dy = f(x)dx, and then integrate on both sides.

$$\sum_{x} y' = 2 \times 45$$

$$\frac{Jy}{4x} = 2 \times 45$$

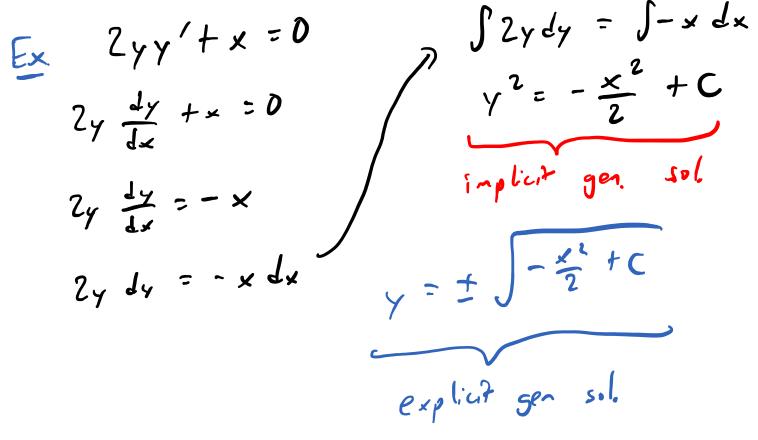
$$\frac{Jy}{4x} = 2 \times 45$$

$$\frac{Jy}{4x} = (2 \times 45) dx$$

$$\int dy = \int (2 \times 45) dx$$

Separable ODEs

 If we can split a first-order ODE so that one side has all of the dependent variable and the other side has all of the independent variable, then we can integrate on both sides.

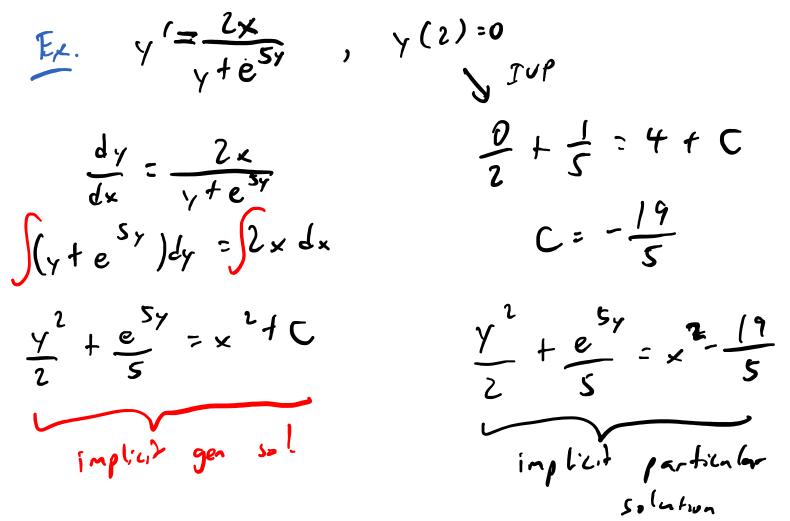


Implicit vs explicit solutions

• An explicit solution to an ODE with independent variable x and dependent variable y is of the form y = f(x).

- An implicit solution to an ODE with independent variable x and dependent variable y is of the form F(x, y) = 0.
 - Sometimes, an implicit solution is the best we can do.
 - Example on next slide.

Implicit solution initial value problem



Try it out: is the following separable?

$$\frac{dy}{dx}$$

$$\cdot y' - y^{2}x \sin x^{2} = 0 \quad \forall ES$$

$$\frac{dy}{dx} = y^{2} \times \sin x^{2} + z^{2}e^{t} = 4 \quad \forall ES$$

$$\frac{dy}{dx} = x^{2} \times \sin x^{2} + z^{2}e^{t} = 4 \quad \forall ES$$

$$\frac{dy}{dt} = x + -t^{2}e^{t}$$

$$\frac{dy}{dt} = x + -t^{2}e^{t}$$

$$\frac{dy}{dt} = x^{2}e^{t} + z^{2}e^{t} + z^{2}e^{t} = 4 \quad \forall ES$$

$$\frac{t^{2}}{dt} = 4 - t^{2}e^{t} + z^{2}e^{t} + z^{2$$

• $y' = \sin(xy)$ NO

A: Separable B: Not separable C: Cannot tell D: ??? E: None of the above

Example: y' = xy, where y(1) = 1dy = xy JUP Jdy = Jxde $|n|| = \frac{1}{2} + C$ $D = \frac{1}{2} + C$ $|x|| = \frac{1}{2}x^2 + C$ $(\frac{1}{2}x^2 - \frac{1}{2})$ $Y^2 e$

Try it out: $y' = 3x^2e^{2y}$ $\frac{d\gamma}{1x} = 3x^2 e^{2\gamma}$ A: $y = -\frac{1}{2}\ln|-2x^3 + C|$ B: $e^{-2y} = -2x^3 + C$ $\frac{dy}{e^{2y}} = 3 \cdot 2 dx$ C: $-\frac{1}{2}e^{-2y} = x^3 + C$ D: All of the above $\int e^{-2\gamma} d\gamma = \int 3 x^2 dx$ E: None of the above $-\frac{1}{2}e^{-2y} = x^{3}+C$ $e^{-2y} = -2x^{3} + c$ -2y = 12 [-2x3/c] y= - 1 1 1-2x31c1