# Pure-time and separable ODEs Lecture 7b: 2023-02-27

MAT A35 – Winter 2023 – UTSC

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# Pure-time ODEs: $y' = f(x)$

- Can solve via integration: $y = \int f(x)dx$

# General vs Particular Solutions

- The general solution has some constant(s) in it, and covers all possible particular solutions.
- A particular solution assigns specific values to those constants.

# Initial value problem

- When you specify “initial conditions”, you choose a single particular solution out of the general solution.

# Try it out

- $\dot{x} = \sin t - 1$
- What is the solution to the initial value problem $x(0) = 0$?

A: $x = \sin t$
B: $x = -\cos t - t$
C: $x = -\cos t - t + 1$
D: $x = -\cos t - t + C$
E: None of the above

# Separating the derivative

- Another way to think about pure-time ODEs:
- We can “split” the derivative $\frac{dy}{dx} = f(x)$ by “multiplying” by $dx$ on both sides: $dy = f(x)dx$, and then integrate on both sides.

# Separable ODEs

- If we can split a first-order ODE so that one side has all of the dependent variable and the other side has all of the independent variable, then we can integrate on both sides.

# Implicit vs explicit solutions

- An explicit solution to an ODE with independent variable $x$ and dependent variable $y$ is of the form $y = f(x)$.

- An implicit solution to an ODE with independent variable $x$ and dependent variable $y$ is of the form $F(x, y) = 0$.
  - Sometimes, an implicit solution is the best we can do.
  - Example on next slide.

# Implicit solution initial value problem

# Try it out: is the following separable?

- $y' - y^2 x \sin x^2 = 0$

- $\dot{x}x^2 + t^2 e^t = 4$

- $\dot{x}t^2 + x^2 e^x = 4$

- $y' = xy$

- $y' = \sin(xy)$

A: Separable
B: Not separable
C: Cannot tell
D: ???
E: None of the above

Example: $y' = xy$, where $y(1) = 1$

# Try it out: $y' = 3x^2 e^{2y}$

A: $y = -\frac{1}{2}\ln|-2x^3 + C|$
B: $e^{-2y} = -2x^3 + C$
C: $-\frac{1}{2}e^{-2y} = x^3 + C$
D: All of the above
E: None of the above