MATA35 - Quiz 1 - Practice

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Problem 1 [10pts]. Solve each of the following problems. If there are multiple potential answers, give your response in the most general form possible. You do not need to show your work.

(a) $\int (x^2 - 4x + 4)$

$$= \frac{1}{3}x^3 - 2x^2 + 4x + C$$

(b) $\int \frac{1}{(y+1)(2y+1)} dy$

$$\frac{1}{(y+1)(2y+1)} = \frac{A}{y+1} + \frac{B}{2y+1} \quad \text{where} \quad A(2y+1) + B(y+1) = 1$$

$$y(2A+B) + (A+B) = 1$$

$$\Rightarrow \begin{cases} 2A+B=0 \\ A+B=1 \end{cases} \begin{cases} B=-2A \\ -A=1 \end{cases} \begin{cases} A=-1 \\ B=2 \end{cases}$$

$$\int \frac{-1}{y+1} dy + \int \frac{2}{2y+1} dy$$

$$= -\ln|y+1| + \ln|2y+1| + C$$

(c) $\int \frac{1}{2}\cos(2x+1)dx$

$$= \frac{1}{4}\sin(2x+1) + C$$

(d) $\int (e^{-2u} + u^3)du$

$$= -\frac{1}{2}e^{-2u} + \frac{1}{4}u^4 + C$$

(e) $\int \left[\frac{d}{dx}((\cos(5 - \sin x))e^{x^2} - 4)\right] dx$

$$= (\cos(5 - \sin x))e^{x^2} + C$$

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Problem 2 [8pts]. Solve each of the following definite integrals. You should simplify as much as possible without a calculator, but may leave answers in terms of $e, \ln, \sin, \sqrt{\dots}$, etc. You must show your work.

(a) $\int_0^{\pi/2} (1+\sin x)^2 \cos x dx$

Let $u = \sin x$

$du = \cos x \, dx$

$$\int_{u=\sin 0 = 0}^{u=\sin\frac{\pi}{2}=1} (1+u)^2 du = \frac{1}{3}(1+u)^3 \Big|_{u=0}^{u=1}$$

$$= \left[\frac{1}{3}(1+1)^3\right] - \left[\frac{1}{3}(1+0)^3\right]$$

$$= \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

(b) $\int_0^{\pi} xe^{-4x} dx + \int_{\pi}^{\infty} xe^{-4x} dx$ $= \int_0^{\infty} xe^{-4x} dx$

Let $u = x$ $\quad v = -\frac{1}{4}e^{-4x}$

$du = dx$ $\quad dv = e^{-4x} dx$

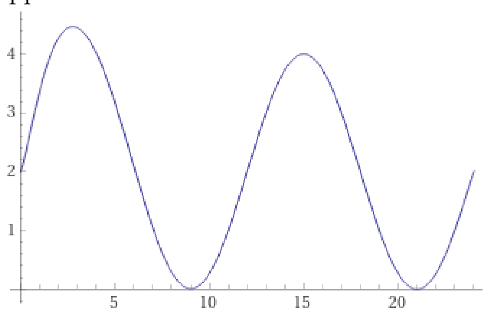
$$= -\frac{x}{4}e^{-4x}\Big|_0^{\infty} - \int_0^{\infty}\left(-\frac{1}{4}e^{-4x}\right)dx = -\frac{x}{4}e^{-4x} + \int \frac{1}{4}e^{-4x} dx$$

$$= -\frac{x}{4}e^{-4x} - \frac{1}{16}e^{-4x}\Big|_0^{\infty}$$

$$= 0 - \left(-\frac{1}{16}\right) = \boxed{\frac{1}{16}}$$

Problem 3 [4pts].

Every day, the Gibson coal-fired power plant in Owensville, Indiana, USA produces about 50 kilotons of CO_2 into the atmosphere. Suppose you set up a post-combustion carbon capture system. Starting from midnight, the instantaneous rate of carbon capture is $f(t) = t^2e^{-t} + 2 + 2\sin\frac{t\pi}{6}$ kilotons/hour of CO_2. After carbon capture, how much CO_2 would be released by the power plant into the air each day? You may approximate $e^{-24} \approx 0$.



Solve $50-\int_0^{24}\left(t^2e^{-t}+2+2\sin\frac{t\pi}{6}\right)dt$ in pieces

$$\int_0^{24} t^2e^{-t}\,dt = \underbrace{-t^2e^{-t}\Big|_0^{24}}_{0} + 2\int_0^{24} te^{-t}\,dt = 2\left[\underbrace{-te^{-t}\Big|_0^{24}}_{0} + \int_0^{24} e^{-t}\,dt\right]$$

$$\boxed{\begin{array}{ll} u=t^2 & v=-e^{-t} \\ du=2t\,dt & dv=e^{-t}dt \end{array}} \qquad \boxed{\begin{array}{ll} u=t & v=-e^{-t} \\ du=dt & dv=e^{-t}dt \end{array}}$$

$$= 2\int_0^{24} e^{-t}\,dt = 2\left[-e^{-t}\Big|_0^{24}\right] = 2\left[0+1\right] = 2$$

$$\int_0^{24} 2\,dt = 2t\Big|_0^{24} = 48$$

$$\int_0^{24} 2\sin\frac{t\pi}{6}\,dt = 2\cdot\frac{6}{\pi}\cdot\left[-\cos\frac{t\pi}{6}\right]\Big|_0^{24} = \frac{12}{\pi}\cdot\underbrace{\left[-\overbrace{\cos 4\pi}^{1} - \left(-\overbrace{\cos 0}^{1}\right)\right]}_{=0}$$

Thus, $50-(2+48+0)=0$ kilotons of CO_2 are released.

Problem 4 [3pts]. 

You are King Arthur's court wizard. Your job is to determine the average airspeed of an unladen African swallow, but you only have one swallow. You decide to send it on a 4-hour round-trip journey, and you measure its speed once an hour, for a total of 5 measurements (including the 0th-hour measurement).

- $v(0) = 15$ miles per hour
- $v(1) = 20$ miles per hour
- $v(2) = 18$ miles per hour
- $v(3) = 23$ miles per hour
- $v(4) = 19$ miles per hour

(a) Approximate the distance traveled using right-rectangular Riemann sums. Use this to compute the average speed.

Sum the last 4 measurements and multiply by $\Delta x = 1$ hour.

Distance traveled $= 20 + 18 + 23 + 19 = 80$ miles.

Avg speed $= \dfrac{80 \text{ miles}}{4 \text{ hours}} = 20$ mph.

(b) Approximate the distance traveled using the trapezoid rule. Use this to compute the average speed.

$$\begin{aligned} 15 \cdot 1 &= 15 \\ 20 \cdot 2 &= 40 \\ 18 \cdot 2 &= 36 \\ 23 \cdot 2 &= 46 \\ 19 \cdot 1 &= 19 \ (+) \\ \hline & \ \ 156 \end{aligned}$$

$\dfrac{156}{2} = 78$ miles.

Avg speed $= \dfrac{78}{4} = 19.5$ mph.

Formulas that may be useful:

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x} \qquad \csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{cos x}$$

$$(\tan x)' = \sec^2 x \qquad (\cot x)' = -\csc^2 x \qquad (\arctan x)' = \frac{1}{1+x^2}$$

$$\sin^2 x + \cos^2 x = 1 \qquad \sin 2x = 2\sin x \cos x \qquad \cos 2x = \cos^2 x - \sin^2 x \qquad \tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2} \qquad \sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$$

$$\cosh\theta = \frac{e^{\theta}+e^{-\theta}}{2} \qquad \sinh\theta = \frac{e^{\theta}-e^{-\theta}}{2}$$

Integration by parts: $\int u\ dv = uv - \int vdu$

Common trignometric values table

θ in Degrees	θ in Radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	0	1	0
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	undefined
180	π	0	-1	0
270	$\frac{3\pi}{2}$	-1	0	undefined
360	2π	0	1	0