# MATA35 - Quiz 3 - Practice Problems

These are practice problems for the take-home quiz. The quiz will be released to Crowdmark on Friday, March 3, at 3pm, as a link to my website. The website will ask you to enter your student number, which will then generate a customized quiz for each student.

Here are a few rules to be aware of:

- You may not collaborate with each other on Quiz 3.
- You may not ask questions on external online forums (such as StackExchange, Kaggle, Chegg, etc.).
- You may ask *public* clarification questions on the MATA35 class Piazza for me or your TAs to answer before 7pm on Friday. This is to ensure that everyone has access to the same information. I will not answer any quiz questions that are asked after 7pm on Friday, or that are not public to the class. Even if you intend to work on the quiz later, make sure you look over the quiz by 7pm in case you have questions.
- You may use the textbook, as well as general online math resources, so long as they are not specific to the problems on Quiz 3. e.g. You may look up a tutorial on how to do linear regression using Python, but if you come across one of the quiz problems on Chegg, you may not use the provided specific solution (indeed, I'd appreciate if you let me know if you encounter the exact quiz question on Chegg).
- You may make full use of calculators (online or physical). However, some problems may ask you to show your work, in which case you cannot simply answer that the calculator solved it for you. Some online calculators and computer algebra systems are extremely powerful and can fully solve some of the quiz problems. However, you should show still your work as if you solved it with only the help of an ordinary scientific calculator. **You must cite every online resource you consult, including online calculators or other sites (e.g. ChatGPT). Failure to cite every online resources you use will be considered an academic integrity violation.**
- All questions should be submitted to Crowdmark. Some questions will ask you to handwrite your answer; make sure your writing is legible. Other questions will ask you to write a report using a Jupyter Notebook / Google Colab. Those should also be uploaded to Crowdmark.
- There will be an ungraded Crowdmark assignment for the practice quiz so you can familiarize yourself with the environment beforehand.
- Be aware that unlike the prior quizzes, Quiz 3 will **not** be basically identical to the practice quizzes with numbers changed. **Make sure you prepare and study accordingly.**

**Problem 1 [2pts].** Consider the function

$$f(x, y) = (x - y)(y - 1)(x + 1) \tag{1}$$

Graph this function at several different zoom levels and describe the behavior of the function in words.

1. **[1pt]** Do you see any local extrema or saddle points? Where are they?
2. **[1pt]** What is the behavior of the function as you go away from the origin in different directions? You should describe the behavior as you go in each direction along the axes, as well as along each of the diagonal directions.

Please include images of the function in your answer, illustrating your description above.

**Problem 2 [3pts].** Compute all of the 1st and 2nd-order partial derivatives of the function $f(x, y)$ from Problem 1 by hand.

**Note**: You must show all of your work for computing the derivatives by hand. You may however use a calculator / online resources to double check your work, which I encourage because Problem 3 depends on the solutions to Problem 2.

**Note**: Make sure you explicitly mark each of your steps. You will not get credit if you do not show your work.

**Problem 3 [6pts].**

1. **[2pts]** Find all critical points of $f(x, y)$ from Problem 1 using the partial derivatives from Problem 2. **Sanity Check**: did you get all the critical points you saw in Problem 1?

2. **[4pts]** Use the various derivative tests (e.g. eigenvalues of the Hessian or the D-test from the book) we learned to classify each critical point as either a minimum, maximum, or saddle point. If it is not possible to classify, explain why.

**Problem 4 [4pts].** Integrate $g(x, y) = \frac{-x^3+2x^2+x}{(y+1)^2}$ over the region bounded by the following inequalities:

- $y \leq -x^2 + 2x$
- $y \geq 0$
- $y \geq x - 1$

Give your answer to 3 decimal places. Show all of your work, but you may use a calculator for evaluating complicated algebraic expressions. Make sure to explain **in words** each of the steps you are doing.

**Note:** You must write out all the work for doing the double integration by hand. You will not get credit if you simply plug in the double integral into an online calculator.

**Note:** the function $g(x, y)$ may be negative in some places, so the interpretation as volume doesn't make as much sense.

**Problem 5 [5pts].**

**For problems 5 and 6, please include your Python code and any generated graphs. Note that some of the answers are qualitiative analyses, with more than one right answer, and they will be marked on the overall quality of your reasoning.**

**On Crowdmark, note that we have combined questions 5 and 6 into a single response to make it easier for you to submit. I also encourage you to additionally submit a link to your code within your PDF report if you used a Google colab, but this is not required.**

Baby names vary in frequency over time. The table below shows the relative frequencies of the names "Holly" and "Henry" as a function of time from the US Social Security records. The numbers are in units of number per 1000 baby girls for "Holly" and number per 1000 baby boys for "Henry".

| Year | Holly | Henry |
|---|---|---|
| 1940 | 0.063 | 5.749 |
| 1945 | 0.284 | 4.811 |
| 1950 | 0.475 | 3.835 |
| 1955 | 1.248 | 2.966 |
| 1960 | 1.240 | 2.293 |
| 1965 | 1.836 | 2.006 |
| 1970 | 2.693 | 1.638 |
| 1975 | 3.235 | 1.282 |
| 1980 | 2.968 | 1.113 |
| 1985 | 3.037 | 1.071 |

1. **[1.25pt]** Use linear regression in Python to find the best fit line for each name.
2. **[1.25pt]** What does your model predict to be the frequencies of each name in the year 1985?
3. **[1.25pt]** Predict the frequencies of each name in the year 2005.
4. **[1.25pt]** Without making reference to the actual values given in problem 6, explain why your predictions are likely to be good or bad.

**Problem 6 [5pts].**

Consider the updated data below.

| Year | Holly | Henry |
|---|---|---|
| 1990 | 1.837 | 1.039 |
| 1995 | 1.325 | 1.253 |
| 2000 | 0.848 | 1.492 |

1. **[1.25pt]** Build a Python model that incorporates the updated data. Note that a linear model may no longer be appropriate, so you may wish to consider using nonlinear regression.
2. **[1.25pt]** Justify your choice of model(s) above.
3. **[1.25pt]** Predict the frequencies of each name in the year 2005 using your model. Note that the actual values are 0.510 and 1.964 for Holly and Henry respectively. How good was your model?
4. **[1.25pt]** Describe some of the remaining shortcomings of your model