

MATA35 - Quiz 5 - Practice

Name: _____

Student ID: _____

Problem 1 [6pts].

(a) Rewrite the complex expression $e^{1+\frac{\pi}{3}i}$ in the standard form $a + bi$, where a and b are real numbers.

(b) Find the modulus and argument (angle) for the following complex number: $-1 - i$.

(c) Find the modulus and argument for $(-1-i)^{1000}$. Simplify as much as is reasonable without a calculator; e.g. you do not need to evaluate things like 2^{500} , but should simplify $\sqrt{2}^{1000} = 2^{500}$.

Problem 2 [5pts]. Consider the following ODE:

$$y''' + y'' - y' - y = x + 1$$

- (a) Find the real homogeneous solution.
- (b) Find a real particular solution
- (c) Find the real general solution

Problem 3 [5pts]. Solve the following initial value problem:

$$\dot{x} = x + 2y$$

$$\dot{y} = 2x + 4y$$

where $x(0) = 5$ and $y(0) = 0$.

Problem 4 [4pts]. Let $\dot{z} = Az$ for each of the following 2x2 matrices A . Classify the equilibrium at the origin by type and stability.

(a) $\begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 4 \\ -1 & 0 \end{bmatrix}$

Problem 5 [5pts]. Tank A contains 100 gallons of water. Tank B contains 200 gallons of water with 50 lb of salt dissolved. Water is pumped from tank A to tank B at a rate of 10 gallons per minute. Water is pumped from tank B to tank A at a rate of 5 gallons per minute. Water is drained from tank B to the outside at a rate of 5 gallons per minute. Pure water is added to tank A at a rate of 5 gallons per minute.

Let $A(t)$ and $B(t)$ denote the total amount of salt present in tanks A and B respectively.

- (a) Draw a 2-compartment model for A and B.
- (b) Write a system of two first-order differential equations modelling the system.
- (c) Find the equilibrium values for A and B.

Formulas that may be useful:

$$\begin{aligned} \tan x &= \frac{\sin x}{\cos x} & \cot x &= \frac{\cos x}{\sin x} & \csc x &= \frac{1}{\sin x} & \sec x &= \frac{1}{\cos x} \\ (\tan x)' &= \sec^2 x & (\cot x)' &= -\csc^2 x & (\arctan x)' &= \frac{1}{1+x^2} \end{aligned}$$

$$\sin^2 x + \cos^2 x = 1 \quad \sin 2x = 2 \sin x \cos x \quad \cos 2x = \cos^2 x - \sin^2 x \quad \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$\begin{aligned} \cos \theta &= \frac{e^{i\theta} + e^{-i\theta}}{2} & \sin \theta &= \frac{e^{i\theta} - e^{-i\theta}}{2i} \\ \cosh \theta &= \frac{e^\theta + e^{-\theta}}{2} & \sinh \theta &= \frac{e^\theta - e^{-\theta}}{2} \end{aligned}$$

Integration by parts: $\int u \, dv = uv - \int v \, du$

Common trigonometric values table

θ in Degrees	θ in Radians	$\sin \theta$	$\cos \theta$	$\tan \theta$
0	0	0	1	0
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	undefined
180	π	0	-1	0
270	$\frac{3\pi}{2}$	-1	0	undefined
360	2π	0	1	0