Problem Set 4 - Due November 18, at 11:59pm

[Your name] and [student ID]
MATB44H3-2019

Problem 1. Let $\dot{x} = Ax(t)$, with $x(0) = x_0$, where A is a 2x2 matrix. Plot the phase portrait for each of the following matrices:

(i) $A = \begin{bmatrix} -1 & 2 \\ 2 & 2 \end{bmatrix}$

(i) $A = \begin{bmatrix} -2 & 2 \\ -1 & 0 \end{bmatrix}$

(i) $A = \begin{bmatrix} -1 & 0 \\ -3 & -4 \end{bmatrix}$

(i) $A = \begin{bmatrix} -3 & 4 \\ -2 & 3 \end{bmatrix}$

Solution. Collaborators: Donald Duck and Mickey Mouse □

Problem 2 (Teschl 3.9). Let $\dot{x} = Ax(t)$, with $x(0) = x_0$. Solve the systems corresponding to the following matrices:

(i) $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}, \qquad x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(i) $A = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}, \qquad x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Solution. Collaborators: Gandalf Greyhame and Bilbo Baggins □

Problem 3 (Teschl 3.10). Solve

$$\dot{x} = -y - t, \qquad \dot{y} = x + t, \qquad x(0) = 1, y(0) = 0$$

Solution. Collaborators: Mark Watney □

Problem 4 (Teschl 3.14). Let A by a real 2-by-2 matrix. Then the eigenvalues can be expressed in terms of the determinant $D = \det(A)$ and the trace $T = \mathrm{tr}(A)$. In particular, (T, D) can take all possible values in $\mathbb{R}^2$ if A ranges over all possible matrices in $\mathbb{R}^{2\times 2}$. Split the (T, D) plane into regions in which the various cases discussed in lecture / Teschl Section 3.2 occur (source, spiral source, sink, spiral sink, saddle, center).

Solution. Collaborators: No one. □