# Department of Computer and Mathematical Sciences, University of Toronto MATB44H3 Differential Equations I

Syllabus—Fall 2019 September 3, 2019

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For general course questions that other students might also have, please use either Piazza or Quercus Discussions so that everyone can see the answer.

For specific questions pertaining only to you, you should email me of course. When you email me, please write "[MATB44]" in the start of the subject line, to minimize the chance I miss your message. Emails *must* be from your utoronto.ca email account (this is a UofT policy), and should contain all necessary information, including your lecture section and tutorial section.

Homepage: http://www.math.toronto.edu/ywyu/MATB44H32019

Quercus: https://q.utoronto.ca/courses/106849

Piazza: https://piazza.com/mail.utoronto.ca/fall2019/matb44h3

Lectures: Tuesdays, 3-5pm in HL B101 and Thursdays, 10-11am in IC 130

**Office hours**: Thursdays, 11am to 1pm in IC 343. If there are many students present, we may move to the common area with chalkboards outside my office. Location is subject to change, especially if there are too many students to fit in those areas.

#### Tutorial

Note that you *must* have a tutorial section. Not only is this a fast-paced class, but we will be doing in-class presentations in the tutorials. Tutorials will start the second week of classes.

<u>Tutorial Number</u>	<b>Time and location</b>	<b>Teaching Assistant</b>
TUT0001	Friday, 9-10am, BV 361	Navin Chandradat
TUT0002	Monday, 9-10am, BV 361	Xiao Jie
TUT0003	Monday, 10-11am, AA 208	Qiaoyu Liang
TUT0004	Thursday, 9-10am, HW 214	Hussein Nassereddine
TUT0005	Tuesday, 6-7pm, AA 208	CheukHung (Gabriel) Lou
TUTooo6	Friday, 10-11am, IC 320	Angela Zavaleta Bernuy
TUT0007	Friday, 12-1pm, SW 143	Navin Chandradat

#### **Overview**

This is an introductory course in differential equations designed for second-year math majors and specialists. However, non-majors/specialists are most welcome, assuming they have the necessary background. Having a heterogeneous student body can make portions of the class more interesting. The prerequisites are MATA36H3/MATA37H3 (single-variable calculus) and MATA22H3/MATA23H3 (linear algebra) and the corequisite is MATB41H3 (multi-variable calculus).

### **Learning Outcomes**

# 1. Computation

You will learn how to solve various types of differential equations. In some cases, we can explicitly write down an analytical solution. In other cases, we cannot write down a solution, but can still understand the general behavior of the solutions. We will focus primarily on first and second order differential equations, as well as linear systems of first order equations. Non-linear equations and difference equations will also be covered.

#### 2. Proof

You will learn to read and understand rigorous mathematical writing and proofs. In particular, Teschl's textbook is quite dense. I will be posting a schedule on the website, and you should at least skim the sections on the schedule before coming to class, and then review it again after we've gone over them in class.

#### 3. Communication

While solving problems and writing proofs may seem like the hard part of mathematics, being able to communicate is just as essential. You will learn how to communicate mathematics to non-expert audiences, in both written and oral form.

#### **Textbooks**

- Gerald Teschl, Ordinary Differential Equations and Dynamical Systems.
  Hard copies should be available from the UTSC bookstore, but a legal online PDF version is also available on the author's website:
  <a href="https://www.mat.univie.ac.at/~gerald/ftp/book-ode/">https://www.mat.univie.ac.at/~gerald/ftp/book-ode/</a>.
  - Teschl is a dense text, but a good introduction to higher-level mathematical writing.
- Morris Tenenbaum and Harry Pollard, *Ordinary Differential Equations*. There are two editions (the original Harper's 1963 publication and a Dover 1985 reprint). They appear to be isomorphic internally. Tenenbaum spends more time on specific problem solving recipes, and has a good number of exercises with solutions that you should try on your own.

# Preliminary course outline (subject to revision)

See the course calendar for an up-to-date schedule and assigned readings. http://www.math.toronto.edu/ywyu/MATB44H32019/#schedule

#### 1. Introduction

- a. What is a differential equation? (Teschl 1.1; Tenenbaum Lesson 1)
- b. Classification of differential equations (Teschl 1.2; Tenenbaum Lessons 2-3)

#### 2. First-order ODEs

(Teschl 1.3-1.6)

- a. Separable ODEs (Tenenbaum Lesson 6)
- b. Homogenous ODEs (Tenenbaum Lesson 7)
- c. ODEs with linear coefficients (Tenenbaum Lesson 8)
- d. Exact ODEs (Tenenbaum Lesson 9)

e. Linear first order ODEs (and integrating factors)

(Tenenbaum Lessons 10-11)

f. Qualitative analysis (direction fields, isoclines, etc.)

(Tenenbaum Lesson 5)

- g. Existence and Uniqueness (Picard-Lindelof) (Teschl 2.1-2.2, 2.7; Tenenbaum Lessons 57-59)
- 3. Second-order ODEs (note that while the books do not always specialize from n<sup>th</sup> to 2<sup>nd</sup> order, we shall)
  - a. Complex Numbers and Complex Functions

(Tenenbaum Lesson 18)

- b. Linear Independence and second-order linear ODEs (Tenenbaum Lesson 19)
- c. Homogeneous second-order linear ODEs (Tenenbaum Lesson 20)
- d. Nonhomogeneous second-order linear ODEs (Tenenbaum Lesson 21-23)
- 4. Systems of first-order ODEs
  - a. Matrix exponential (Teschl 3.1)
  - b. Linear autonomous first-order systems (Teschl 3.2)
  - c. General linear first-order systems (Teschl 3.4, Tenenbaum Lesson 31)
  - d. Existence and Uniqueness for system of first-order ODEs (Tenenbaum Lesson 62)
  - e. Determinants and Wronksians (Tenenbaum Lessons 63-64)
  - f. Series methods

(Tenenbaum Lessons 37-40)

- 5. Miscellaneous topics (as time permits)
  - a. Dynamical systems

(Teschl 6.1-6.5)

b. Discrete dynamical systems (Teschl 10.1-10.4)

c. ..

## Lectures, Notes, and WebOption

I will be posting the topics for upcoming lectures on the schedule online. I strongly recommend that you at least skim the listed sections in the textbooks before coming to class, and then follow-up by reading them in depth after class. Schedule:

http://www.math.toronto.edu/ywyu/MATB44H32019/#schedule

All lectures will be recorded and broadcast online for 1 week following the lecture. This is not meant to substitute for coming to class, but to supplement your studying and as a safety net in case you have to miss class: <a href="https://lecturecast.utsc.utoronto.ca/login.php">https://lecturecast.utsc.utoronto.ca/login.php</a>

Thanks to the wonders of technology, all of my notes will be available online, almost as soon as I write them in class. There's a delay of about 10-30 seconds when I write something, but you

should feel free to follow online on your own computers, especially to look at stuff I wrote much earlier in class: http://www.math.toronto.edu/vwyu/MATB44H32019/notes

#### **Evaluation and Grades**

• 40%: 3 hour final exam during final exam period

Note that you must pass the final to pass the class.

• 24%: Two 50-minute in-class midterm exams

Midterm 1: Thursday, October 3, in class

Midterm 2: Thursday, November 7, in class

Each midterm exam will be worth 12% of your final mark, and will take place in class. Information about what the exams will cover will be posted at least one week before the exams. **There will be no make-up exams.** If you have a university documented accommodation that causes you to miss an exam, we will use your other midterm mark as the grade for the missed exam. **If you do not, then you will receive a o.** Note that enrollment in another class that overlaps with this one is **not** an acceptable reason for missing a midterm, and you will receive a o.

**ADDENDUM:** We will allow you to shift up to 4% of your final mark from one midterm to the other. i.e. you may optionally make one midterm worth 16% of your final mark and the other midterm worth 8%. Since it is always purely beneficial for you to put 16% of your final mark on the midterm on which you performed better and 8% on the one on which you did worse, we will assume that you wish to do so and automatically make this adjustment for you, unless you object in writing.

• 10%: Math communication

This is a partnered activity. Your partner should be chosen from someone who is in the same tutorial. If logistical issues occur (e.g. your partner drops the class), you may either choose to continue as a singleton, or form a new partnership with another singleton. With the permission of the instructor, you may also form a group of 3, but there will be a larger work requirement assigned that may be modified on a case by case basis.

The two of you should choose some differential equation or system of differential equations that you find fascinating. Your task is then to convince a disinterested audience why they too should find that topic interesting. You may choose either an audience of your peers (fellow students in class) or a lay audience (of people whose last math class was in high school). So long as a cogent argument is made, you may use any kind of reasoning, ranging from mathematical elegance to usefulness in the real world.

o 1%: Topic choice

You and your partner should write a 200 word description of why your chosen topic is interesting. This is due by October 25. If the choice of topic is inappropriate, we will let you know within a week, and you must choose a new topic by November 5.

4%: Oral presentation.

You and your partner must give a 3-5-minute oral presentation on your chosen topic. One of the tutorials near the end of the term will be allocated for in-class presentations. Alternately, you may choose to create a 3-5-minute video instead for submission to Quercus.

o 5%: Term paper due on November 28.

The term paper should be between 1,000 to 2,000 words long. We

recommend the paper be electronically typeset in LaTeX. However, if you choose to target a lay audience, you need not use LaTeX and may use any word processing software or may neatly handwrite the document and scan it to PDF. All events, the final document should be uploaded to Quercus as a PDF. The paper should include citations and figures where relevant.

• 10%: Online guizzes on Quercus/WebWork.

We will regularly be posting online quizzes on Quercus/WebWork. You will always have at least two weeks to complete the quiz, from the date of posting. The online quizzes will be relatively straight-forward applications of techniques learned in class, ensuring that you are able to perform computation. The grading will be all-or-nothing. You must correctly answer all questions on a quiz, or else you will receive a 0% on that quiz. However, you may repeat any quiz as many times as you wish before the deadline. The questions on the quizzes will be randomly generated, so you likely will not encounter the exact same question twice, but you should expect to see the same classes of questions repeatedly.

You may not collaborate while taking a quiz. This is intended to test your mastery of basic principles. However, you are allowed (indeed, encouraged!) to seek help from others if you fail a quiz repeatedly, and you can bring the questions you missed to friends/TAs for guidance. Once you feel ready, you should of course take the quiz again on your own without outside help.

• 16%: Homework problem sets.

There will be 5 problem sets as homework assignments, due dates to be announced later. Your lowest problem set mark will be automatically dropped. i.e. your final homework grade will be the average of your 4 top scoring marks.

Collaboration is encouraged on homework assignments. However, there are two caveats. First, attribution is important; therefore, you must prominently write down all sources you consulted and the names of all your collaborators on the problem set to give credit where it's due---additionally, we strongly discourage working in groups of larger than 3-4 for any particular problem. Second, it is important that you make sure you fully understand the material yourself. As such, you may solve the problem together, but you have to write up the final answer yourself. Ideally, if solving the problem was a group effort, you write up the final paper without looking at your notes from the group study session. This ensures that you actually understand the solution.

Homework solutions should be uploaded to either Quercus or Crowdmark (as instructed) by the due date. We recommend typesetting in LaTeX (and will provide a guide and template document), but you may also scan a handwritten assignment, though you must ensure that all the writing is legible.

## Late policy / extension penalty

Extensions are automatically given for homework problem sets, but a penalty will apply. An additive penalty of 10% per day will apply to every late assignment. E.g. if an assignment is due October 1<sup>st</sup>, but is turned in two days late on October 3<sup>rd</sup> by 11:59pm, and receives a mark of 90%, after penalties the mark will 70%. If it is turned in a full week late, the mark after penalties will be 20%. And if any assignment is more than 10 days late, you will receive no credit for the assignment. Note that the penalty is applied *before* dropping the lowest problem set mark.

There will be no extensions for the online quizzes. There is already a long window for the online quizzes, so we strongly recommend you do not wait until the last day.

There will be no extensions given for the math communication project components. This is largely a practical matter, as the due date for the project is the last day of classes, and the University of Toronto prohibits setting assignments due past the end of term. In exceptional cases, you may petition UTSC for longer extensions (see section VII.1. in the UTSC Handbook).

# Laptop / cell phone policy

I encourage the use of technology in the classroom. In particular, as the instructor's notes will appear at <a href="http://www.math.toronto.edu/ywyu/MATB44H32019/notes">http://www.math.toronto.edu/ywyu/MATB44H32019/notes</a> immediately, you are encouraged to refer back to them. Additionally, we will be using Mentimeter interactive presentation software to allow student engagement, which will also be done on your personal laptop or cell phone (this is not required, but strongly encouraged).

However, please stay on task when using laptops / cell phones. Not only will checking social media, texting, or other non-course specific activities detract from your learning experience (and negatively affect your eventual grade!), but it also may be distracting for your classmates. E.g. if you are playing video games in class (which I strongly discourage), please sit in the back row to avoid disturbing others. I reserve the right to remove you from class if you are being disruptive.

## Piazza / Quercus Discussions

If you have a question, chances are someone else in your class does too! As such, for general questions, we prefer that you communicate through the class forums online. Sometimes, other students will be able to answer your question. The teaching staff will also regularly check the online forums, and this will ensure that your classmates can also read the answer.

Piazza: https://piazza.com/mail.utoronto.ca/fall2019/matb44h3

Quercus: <a href="https://q.utoronto.ca/courses/106849">https://q.utoronto.ca/courses/106849</a>

#### **Quercus Info**

This course uses the University's learning management system, Quercus, to post information about the course. This includes posting readings and other materials required to complete class activities and course assignments, as well as sharing important announcements and updates. The site is dynamic and new information and resources will be posted regularly as we move through the term, so please make it a habit to log in to the site on a regular, even daily, basis. To access the course website, go to the U of T Quercus log-in page at <a href="https://q.utoronto.ca">https://q.utoronto.ca</a>. Once you have logged in to Quercus using your UTORid and password, you should see the link or "card" for MATB44H3. You may need to scroll through other cards to find this. Click on the MATB44H3 link to open our course area, view the latest announcements and access your course resources. There are Quercus help guides for students that you can access by clicking on the "?" icon in the left side column. SPECIAL NOTE ABOUT GRADES POSTED ONLINE: Please also note that any grades posted are for your information only, so you can view and track your progress through the course. No grades are considered official, including any posted in Quercus at any point in the term, until they have been formally approved and posted on ACORN at the end of the course. Please contact me as soon as possible if you think there is an error in any grade posted on Quercus.

#### **Accessibility Statement**

Students with diverse learning styles and needs are welcome in this course. In particular, if you have a disability/health consideration that may require accommodations, please feel free to approach me and/or the Access Abillity Services as soon as possible.

Accessibility Services staff (located in Rm SW302, Science Wing) are available by appointment to assess specific needs, provide referrals and arrange appropriate accommodations. You can reach them by phone at 416-287-7560 or email <a href="mailto:ability@utsc.utoronto.ca">ability@utsc.utoronto.ca</a>. The sooner you let us know your needs, the quicker we can assist you in achieving your learning goals in this course.

# **Specific Medical Circumstances**

If you become ill and it affects your ability to do your academic work, consult the course instructor right away. Normally, you will be asked for medical documentation in support of your specific medical circumstances. The University's Verification of Student Illness or Injury (VOI) form is recommended because it indicates the impact and severity of the illness, while protecting your privacy about the details of the nature of the illness. You can submit a different form (like a letter from the doctor), as long as it is an original document, and it contains the same information as the VOI. For more information, please see <a href="http://www.illnessverification.utoronto.ca/">http://www.illnessverification.utoronto.ca/</a>

If you get a concussion, break your hand, or suffer some other acute injury, you should register with Accessibility Services as soon as possible. A student registered with the AS isn't usually asked to provide a VOI because registration with AS already requires students to provide health-related documentation.

 $(pg.23\ \underline{http://www.viceprovoststudents.utoronto.ca/Assets/Students+\underline{Digital+Assets/Demystifying+Academic+Accommodations.pdf})$ 

#### **Religious Accommodation**

The University has a commitment concerning accommodation for religious observances. I will make every reasonable effort to avoid scheduling tests, examinations, or other compulsory activities on religious holy days not captured by statutory holidays. According to University Policy, if you anticipate being absent from class or missing a major course activity (like a test, or in-class assignment) due to a religious observance, please let me know as early in the course as possible, and with sufficient notice (at least two to three weeks), so that we can work together to make alternate arrangements.

## **Academic Integrity**

The University treats cases of cheating and plagiarism very seriously. The University of Toronto's Code of Behaviour on Academic Matters

(http://www.governingcouncil.utoronto.ca/policies/behaveac.htm) outlines the behaviours that constitute academic dishonesty and the processes for addressing academic offences. Potential offences in papers and assignments include using someone else's ideas or words without appropriate acknowledgement, submitting your own work in more than one course without the permission of the instructor, making up sources or facts, obtaining or providing unauthorized assistance on any assignment. On tests and exams cheating includes using or possessing unauthorized aids, looking at someone else's answers during an exam or test, misrepresenting your identity, or falsifying or altering any documentation required by the University, including (but not limited to) doctor's notes.

# **Privacy/FIPPA Statement**

Personal information is collected pursuant to section 2(14) of the University of Toronto Act, 1971, and at all times it will be protected in accordance with the Freedom of Information and Protection of Privacy Act. Please note that this course requires presentations of one's work to the group. For more information, please refer to <a href="https://www.utoronto.ca/privacy">www.utoronto.ca/privacy</a>.

# Harassment/Discrimination

The University of Toronto is a richly diverse community and as such is committed to providing an environment free of any form of harassment, misconduct, or discrimination. In this course, I seek to foster a civil, respectful, and open-minded climate in which we can all work together to develop a better understanding of key questions and debates through meaningful dialogue. As such, I expect all involved with this course to refrain from actions or behaviours that intimidate, humiliate, or demean persons or groups or that undermine their security or self-esteem based on traits related to race, religion, ancestry, place of origin, colour, ethnic origin, citizenship, creed, sex, sexual orientation, gender identity, gender expression, age, marital status, family status, disability, receipt of public assistance or record of offences.