1.4a Higher order equations

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Let's review linear ODEs:

$$a_k(t)\frac{d^k}{dt^k}x + \cdots + a_1(t)\frac{dx}{dt} + a_0(t)x = b(t)$$

(1) Find k linearly ind. sols to homogeneous

$$a_k(t)\frac{d^k x}{dt^k} + \cdots + a_1(t)\frac{dx}{dt} + a_0(t)x = 0$$

called $x_1(t), x_2(t), \dots, x_k(t)$

$x_h(t) = \sum_{i=1}^{k} c_i x_i(t)$ $\forall c_i \in \mathbb{R}$ is a sol. to hom.

- If $a_i(t) \equiv a_i$ constants, then we can use the characteristic eqn
$$a_k \lambda^k + \cdots + a_1\lambda + a_0 = 0.$$

Then $t^{\beta} e^{\lambda t}$ is a sol. to hom. eqn, $\beta = \{0, \dots, m(\lambda)-1\}$ $m(\lambda)$ = multiplicity of λ as a solution to char. eqn.

(2) Find any sol. $x_p(t)$ to inhog eqn.

- Method of undetermined coeff.
- Variation of parameters

(3) Gen sol. $x(t) = x_h(t) + x_p(t)$ by Principle of Superposition

Linear difference equations

$$a_k(t)x(t+k) + \cdots + a_1(t)x(t+1) + a_0(t)x(t) = b(t)$$

(1) Find k lin. ind. sol. to homogeneous

$$a_k(t)x(t+k) + \cdots + a_1(t)x(t+1) + a_0(t)x(t) = 0$$

called $x_1, x_2, \dots, x_k$.

$x_h = \sum_{i=1}^{k} c_i x_i$ $\forall c_i \in \mathbb{R}$ is a sol. to hom. eqn.

- If $a_i(t) \equiv a_i$ constants, then we can use the characteristic eqn
$$a_k \lambda^k + \cdots + a_1\lambda + a_0 = 0.$$

Then $t^{\beta}\lambda^{t}$ is a sol. to the hom. eqn, $\beta = \{0, \dots, m(\lambda)-1\}$ $m(\lambda)$ = multiplicity of λ as a solution to char. eqn.

(2) Find any sol x_p to inhog eqn

- Method of undetermined coeff.
- Variation of parameters

(3) Gen sol $x = x_h + x_p$ by Principle of Superposition

Def. 1.7 The zeros $\lambda_1, \dots, \lambda_k$ of the char eqn $a_k\lambda^k + \cdots + a_1\lambda + a_0 = 0$ are often called eigenvalues. If for some i and for all j,

$$|\lambda_i| \geq |\lambda_j|,$$ then λ_i is a dominant eigenvalue

or, $$|\lambda_i| > |\lambda_j|,$$ then λ_i is the strictly dom. eigenvalue.

Asymptotically, the behavior of a typical solution is controlled by the dominant eigenvalue, e.g. if λ_i is a dominant eigenvalue, dominant eigenvalue. e.g. if λ_i is a dominant eigenvalue, and $|\lambda_i| < 1$, then solutions converge to 0.